12

Vapour Power Cycles

12.1. CARNOT CYCLE

Figure 12.1 shows a Carnot cycle on *T-s* and *p-V* diagrams. It consists of (*i*) two constant pressure operations (4-1) and (2-3) and (*ii*) two frictionless adiabatics (1-2) and (3-4). These operations are discussed below :

1. **Operation (4-1).** 1 kg of boiling water at temperature T_1 is heated to form wet steam of dryness fraction x_1. Thus heat is absorbed at constant temperature T_1 and pressure p_1 during this operation.

2. **Operation (1-2).** During this operation steam is expanded isentropically to temperature T_2 and pressure p_2. The point '2' represents the condition of steam after expansion.

3. **Operation (2-3).** During this operation heat is rejected at constant pressure p_2 and temperature T_2. As the steam is exhausted it becomes wetter and cooled from 2 to 3.

4. **Operation (3-4).** In this operation the wet steam at '3' is compressed isentropically till the steam regains its original state of temperature T_1 and pressure p_1. Thus cycle is completed.

Refer *T-s* diagram :

Heat supplied at constant temperature T_1 [operation (4-1)] = area 4-1-*b*-*a* = T_1 $(s_1 - s_4)$ or T_1 $(s_2 - s_3)$.

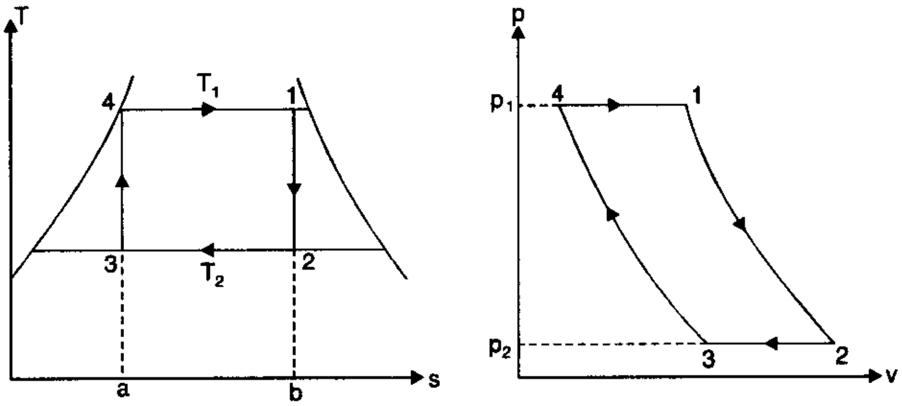


Fig. 12.1. Carnot cycle on *T-s* and *p-V* diagrams.

Heat rejected at constant temperature T_2 (operation 2-3) = area 2-3-*a*-*b* = T_2 $(s_2 - s_3)$.

Since there is no exchange of heat during isentropic operations (1-2) and (3-4)

Net work done = Heat supplied – heat rejected

$$= T_1 (s_2 - s_3) - T_2 (s_2 - s_3)$$

$$= (T_1 - T_2)(s_2 - s_3).$$

$$\text{Carnot cycle } \eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{(T_1 - T_2)(s_2 - s_3)}{T_1 (s_2 - s_3)} = \frac{T_1 - T_2}{T_1} \quad \text{...(12.1)}$$

Limitations of Carnot Cycle

Though Carnot cycle is simple (thermodynamically) and has the *highest thermal efficiency* for given values of T_1 and T_2, yet it is *extremely difficult to operate in practice* because of the following *reasons* :

1. It is difficult to compress a wet vapour isentropically to the saturated state as required by the process 3-4.

2. It is difficult to control the quality of the condensate coming out of the condenser so that the state '3' is exactly obtained.

3. The efficiency of the Carnot cycle is greatly affected by the temperature T_1 at which heat is transferred to the working fluid. Since the critical temperature for steam is only 374°C, therefore, if the cycle is to be operated in the *wet region*, the maximum possible temperature is severely limited.

4. The cycle is still more difficult to operate in practice with superheated steam due to the necessity of supplying the superheat at constant temperature instead of constant pressure (as it is customary).

- *In a practical cycle, limits of pressure and volume are far more easily realised than limits of temperature so that at present no practical engine operates on the Carnot cycle, although all modern cycles aspire to achieve it.*

12.2. RANKINE CYCLE

Rankine cycle *is the theoretical cycle on which the steam turbine (or engine) works.*

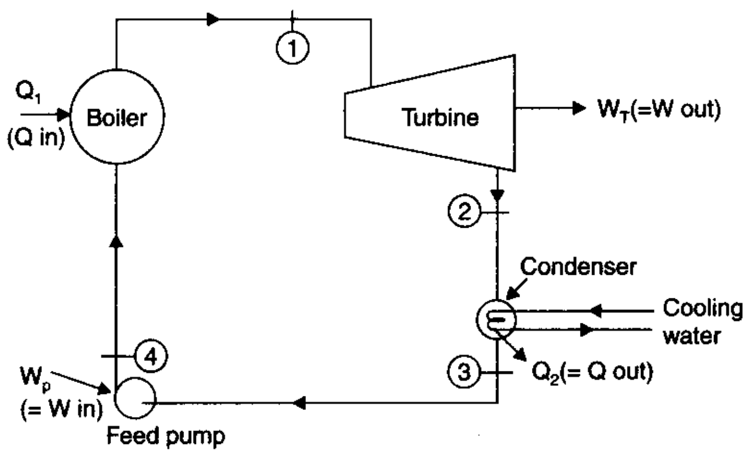


Fig. 12.2. Rankine cycle.

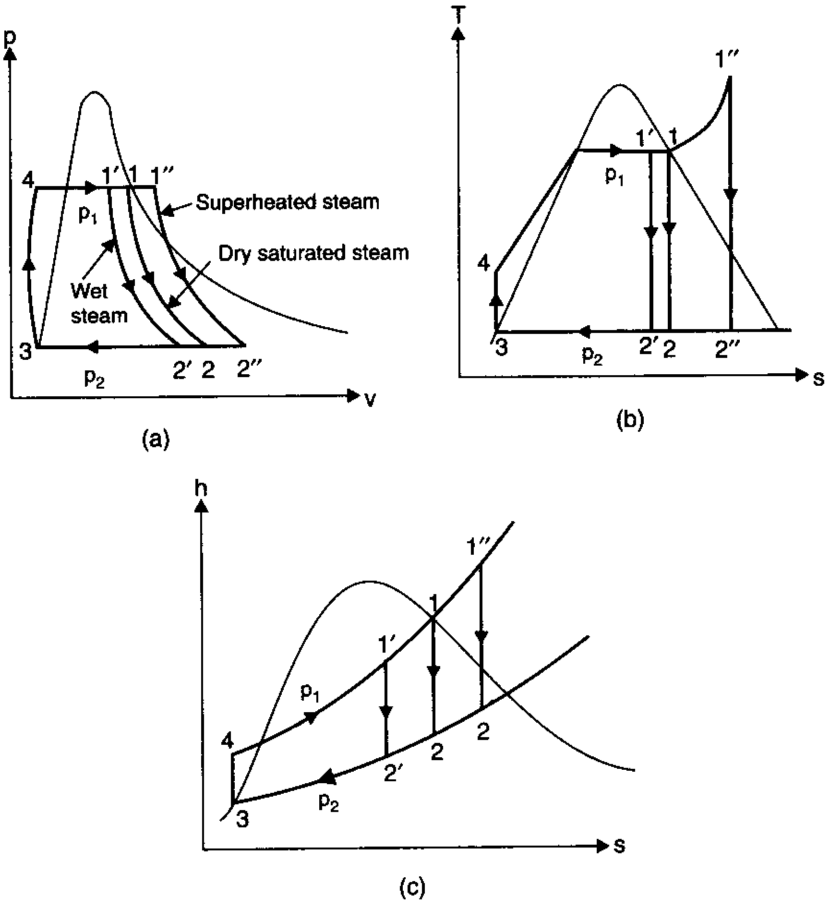


Fig. 12.3. (*a*) *p-v* diagram ; (*b*) *T-s* diagram ; (*c*) *h-s* diagram for Rankine cycle.

The Rankine cycle is shown in Fig. 12.2. It comprises of the following *processes* :

Process 1-2 : Reversible adiabatic expansion in the turbine (or steam engine).

Process 2-3 : Constant-pressure transfer of heat in the condenser.

Process 3-4 : Reversible adiabatic pumping process in the feed pump.

Process 4-1 : Constant-pressure transfer of heat in the boiler.

Fig. 12.3 shows the Rankine cycle on *p-v*, *T-s* and *h-s* diagrams (when the saturated steam enters the turbine, the steam can be wet or superheated also).

Considering 1 kg of fluid :

Applying *steady flow energy equation* (S.F.E.E.) to boiler, turbine, condenser and pump :

(*i*) **For boiler** (as control volume), we get

$$h_{f_4} + Q_1 = h_1$$

$$\therefore \quad Q_1 = h_1 - h_{f_4} \quad \text{...(12.2)}$$

(*ii*) **For turbine** (as control volume), we get

$$h_1 = W_T + h_2, \text{ where } W_T = \text{turbine work}$$

$$\therefore \quad W_T = h_1 - h_2 \quad \text{...(12.3)}$$

(*iii*) **For condenser**, we get

$$h_2 = Q_2 + h_{f_3}$$

$$\therefore \quad Q_2 = h_2 - h_{f_3} \quad ...(12.4)$$

(*iv*) **For the feed pump**, we get

$$h_{f_3} + W_P = h_{f_4}, \quad \text{where, } W_P = \text{Pump work}$$

$$\therefore \quad W_P = h_{f_4} - h_{f_3}$$

Now, efficiency of Rankine cycle is given by

$$\eta_{\text{Rankine}} = \frac{W_{\text{net}}}{Q_1} = \frac{W_T - W_P}{Q_1}$$

$$= \frac{(h_1 - h_2) - (h_{f_4} - h_{f_3})}{(h_1 - h_{f_4})} \quad ...(12.5)$$

The feed pump handles liquid water which is incompressible which means with the increase in pressure its density or specific volume undergoes a little change. Using general property relation for reversible adiabatic compression, we get

$$Tds = dh - vdp$$

$$\because \quad ds = 0$$

$$\therefore \quad dh = vdp$$

or $$\Delta h = v\,\Delta p \quad \text{...... (since change in specific volume is negligible)}$$

or $$h_{f_4} - h_{f_3} = v_3\,(p_1 - p_2)$$

When p is in bar and v is in m^3/kg, we have

$$h_{f_4} - h_{f_3} = v_3\,(p_1 - p_2) \times 10^5 \text{ J/kg}$$

The feed pump term ($h_{f_4} - h_{f_3}$) being a small quantity in comparison with turbine work, W_T, is usually neglected, *especially when the boiler pressures are low*.

Then, $$\eta_{\text{Rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_4}} \quad ...[12.5\ (a)]$$

Comparison between Rankine Cycle and Carnot Cycle

The following points are worth noting :

(*i*) Between the same temperature limits Rankine cycle provides a higher specific work output than a Carnot cycle, consequently Rankine cycle *requires a smaller steam flow rate resulting in smaller size plant for a given power output*. However, Rankine cycle calls for higher *rates of heat transfer in boiler and condenser*.

(*ii*) Since in Rankine cycle only part of the heat is supplied isothermally at constant higher temperature T_1, therefore, its *efficiency is lower* than that of Carnot cycle. The efficiency of the Rankine cycle will approach that of the Carnot cycle more nearly if the *superheat temperature rise is reduced*.

(*iii*) The advantage of using pump to feed liquid to the boiler instead to compressing a wet vapour is obvious that the *work for compression is very large compared to the pump*.

Fig. 12.4 shows the plots between efficiency and specific steam consumption against boiler pressure for Carnot and ideal Rankine cycles.

Effect of Operating Conditions on Rankine Cycle Efficiency

The Rankine cycle efficiency can be *improved* by :

(*i*) *Increasing the average temperature at which heat is supplied.*

(*ii*) *Decreasing/reducing the temperature at which heat is rejected.*

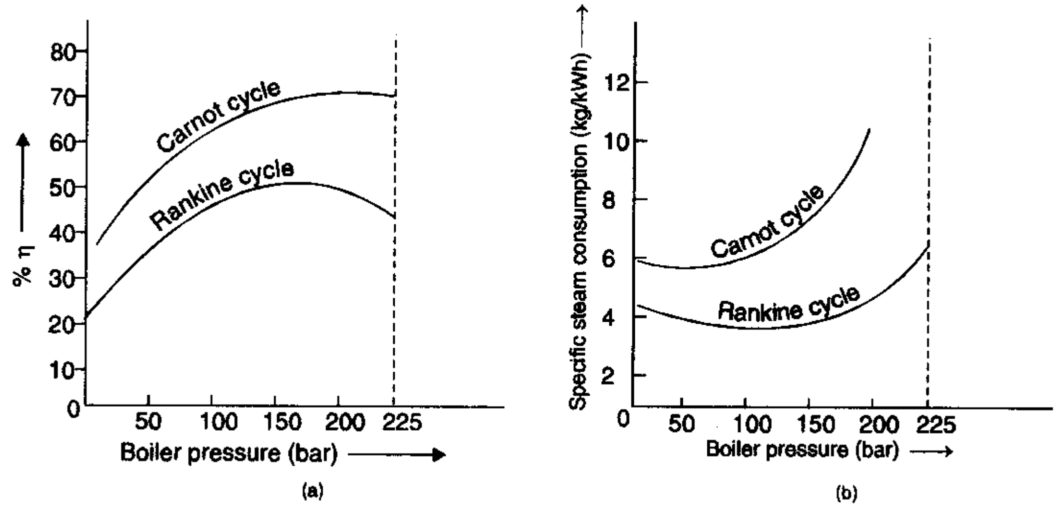


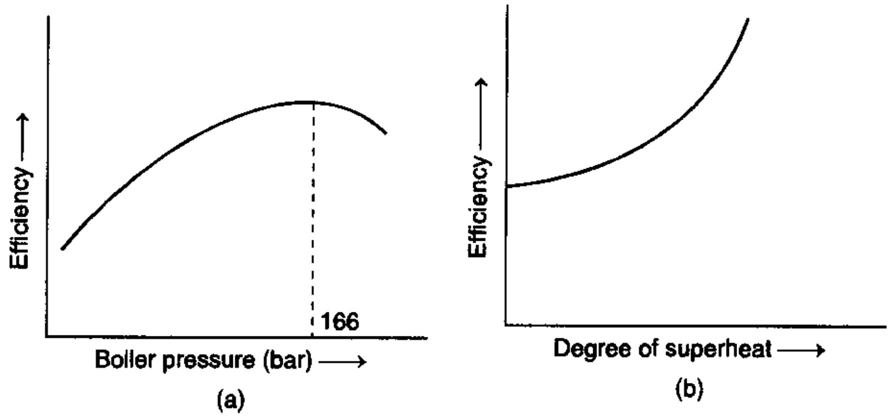
Fig. 12.4

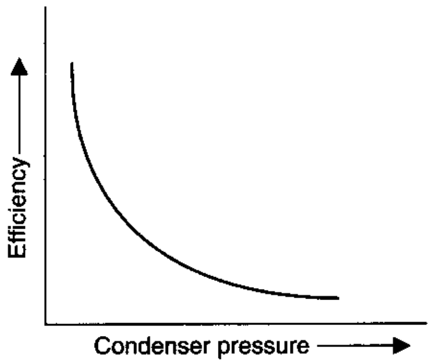
This can be achieved by making suitable changes in the conditions of steam generation or condensation, as discussed below :

1. ***Increasing boiler pressure.*** It has been observed that by increasing the boiler pressure (other factors remaining the same) the cycle tends to rise and reaches a maximum value at a boiler pressure of about 166 bar [Fig. 12.5 (*a*)].

2. ***Superheating.*** All other factors remaining the same, if the steam is superheated before allowing it to expand the Rankine cycle efficiency may be increased [Fig. 12.5 (*b*)]. The use of superheated steam also ensures longer turbine blade life because of the absence of erosion from high velocity water particles that are suspended in wet vapour.

3. ***Reducing condenser pressure.*** The thermal efficiency of the cycle can be amply improved by reducing the condenser pressure [Fig. 12.5 (*c*)] (hence by reducing the temperature at which heat is rejected), especially in high vacuums. But the increase in efficiency is obtained at the *increased cost of condensation apparatus.*





(c)

Fig. 12.5. Effect of operating conditions on the thermal efficiency of the Rankine cycle.

The thermal efficiency of the Rankine cycle is also *improved* by the following methods :

(*i*) *By regenerative feed heating.*

(*ii*) *By reheating of steam.*

(*iii*) *By water extraction.*

(*iv*) *By using binary-vapour.*

☞**Example 12.1.** *The following data refer to a simple steam power plant :*

S. No.	*Location*	*Pressure*	*Quality/temp.*	*Velocity*
1.	*Inlet to turbine*	*6 MPa (= 60 bar)*	*380°C*	—
2.	*Exit from turbine inlet to condenser*	*10 kPa (= 0.1 bar)*	*0.9*	*200 m/s*
3.	*Exit from condenser and inlet to pump*	*9 kPa (= 0.09 bar)*	*Saturated liquid*	—
4.	*Exit from pump and inlet to boiler*	*7 MPa (= 70 bar)*	—	—
5.	*Exit from boiler*	*6.5 MPa (= 65 bar)*	*400°C*	—
	Rate of steam flow = 10000 kg/h.			

Calculate :

(*i*) *Power output of the turbine.*

(*ii*) *Heat transfer per hour in the boiler and condenser separately.*

(*iii*) *Mass of cooling water circulated per hour in the condenser. Choose the inlet temperature of cooling water 20°C and 30°C at exit from the condenser.*

(*iv*) *Diameter of the pipe connecting turbine with condenser.*

Solution. Refer Fig. 12.6.

(*i*) **Power output of the turbine, P :**

At 60 bar, 380°C : From steam tables,

$$h_1 = 3043.0 \text{ (at 350°C)} + \frac{3177.2 - 3043.0}{(400 - 350)} \times 30 \text{ ... By interpolation}$$

$$= 3123.5 \text{ kJ/kg}$$

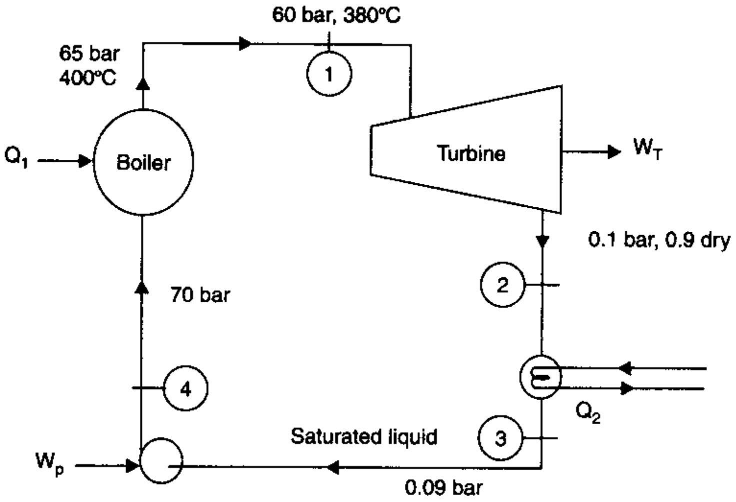


Fig. 12.6

At 0.1 bar :

h_{f_2} = 191.8 kJ/kg, h_{fg_2} = 2392.8 kJ/kg (from steam tables)

and x_2 = 0.9 (given)

$\therefore$ $h_2 = h_{f_2} + x_2\, h_{fg_2}$ = 191.8 + 0.9 × 2392.8 = 2345.3 kJ/kg

Power output of the turbine = $m_s\,(h_1 - h_2)$ kW,

[where m_s = Rate of steam flow in kg/s and h_1, h_2 = Enthalpy of steam in kJ/kg]

$$= \frac{10000}{3600}\,(3123.5 - 2345.3) = 2162 \text{ kW}$$

Hence power output of the turbine = **2162 kW. (Ans.)**

(*ii*) **Heat transfer per hour in the boiler and condenser :**

At 70 bar : h_{f_4} = 1267.4 kJ/kg

At 65 bar, 400°C : $h_a = \dfrac{3177.2\,(60\text{ bar}) + 3158.1\,(70\text{ bar})}{2}$ = 3167.6 kJ/kg

......(By interpolation)

$\therefore$ *Heat transfer per hour in the boiler,*

$Q_1 = 10000\,(h_a - h_{f_4})$ kJ/h

= 10000 (3167.6 − 1267.4) = **1.9 × 10⁷ kJ/h. (Ans.)**

At 0.09 bar : h_{f_3} = 183.3 kJ/kg

Heat transfer per hour in the condenser,

$Q_1 = 10000\,(h_2 - h_{f_3})$

= 10000 (2345.3 − 183.3) = **2.16 × 10^7 kJ/h. (Ans.)**

(*iii*) **Mass of cooling water circulated per hour in the condenser, m_w :**

Heat lost by steam = Heat gained by the cooling water

$$Q_2 = m_w \times c_{pw}\,(t_2 - t_1)$$

$$2.16 \times 10^7 = m_w \times 4.18\,(30 - 20)$$

$$\therefore \quad \mathbf{m_w} = \frac{2.16 \times 10^7}{4.18\,(30 - 20)} = \mathbf{1.116 \times 10^7 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

(*iv*) **Diameter of the pipe connecting turbine with condenser, d :**

$$\frac{\pi}{4} d^2 \times C = m_s x_2 v_{g_2} \quad ...(i)$$

Here, d = Diameter of the pipe (m),

C = Velocity of steam = 200 m/s (given),

m_s = Mass of steam in kg/s,

x_2 = Dryness fraction at '2', and

v_{g_2} = Specific volume at pressure 0.1 bar (= 14.67 m³/kg).

Substituting the various values in eqn. (*i*), we get

$$\frac{\pi}{4} d^2 \times 200 = \frac{10000}{3600} \times 0.9 \times 14.67$$

$$d = \left(\frac{10000 \times 0.9 \times 14.67 \times 4}{3600 \times \pi \times 200}\right)^{1/2} = 0.483 \text{ m or } \mathbf{483 \text{ mm.}} \quad \textbf{(Ans.)}$$

Example 12.2. *In a steam power cycle, the steam supply is at 15 bar and dry and saturated. The condenser pressure is 0.4 bar. Calculate the Carnot and Rankine efficiencies of the cycle. Neglect pump work.*

Solution. Steam supply pressure, $p_1 = 15$ bar, $x_1 = 1$

Condenser pressure, $p_2 = 0.4$ bar

Carnot and Rankine efficiencies :

From steam tables :

At 15 bar : $t_s = 198.3°C$, $h_g = 2789.9$ kJ/kg, $s_g = 6.4406$ kJ/kg K

At 0.4 bar : $t_s = 75.9°C$, $h_f = 317.7$ kJ/kg, $h_{fg} = 2319.2$ kJ/kg,

$s_f = 1.0261$ kJ/kg K, $s_{fg} = 6.6448$ kJ/kg K

$$T_1 = 198.3 + 273 = 471.3 \text{ K}$$

$$T_2 = 75.9 + 273 = 348.9 \text{ K}$$

$$\therefore \quad \eta_{carnot} = \frac{T_1 - T_2}{T_1} = \frac{471.3 - 348.9}{471.3}$$

$$= \mathbf{0.259 \text{ or } 25.9\%.} \quad \textbf{(Ans.)}$$

$$\eta_{Rankine} = \frac{\text{Adiabatic or isentropic heat drop}}{\text{Heat supplied}} = \frac{h_1 - h_2}{h_1 - h_{f_2}}$$

where $h_2 = h_{f_2} + x_2 h_{fg_2} = 317.7 + x_2 \times 2319.2$ ...(*i*)

Value of x_2 :

As the steam expands isentropically,

$$\therefore \quad s_1 = s_2$$

$$6.4406 = s_{f_2} + x_2 s_{fg_2} = 1.0261 + x_2 \times 6.6448$$

$$\therefore \quad x_2 = \frac{6.4406 - 1.0261}{6.6448} = 0.815$$

$$\therefore \quad h_2 = 317.7 + 0.815 \times 2319.2 = 2207.8 \text{ kJ/kg} \quad \text{[From eqn. } (i)]$$

Hence, $$\eta_{Rankine} = \frac{2789.9 - 2207.8}{2789.9 - 317.7} = \mathbf{0.2354 \text{ or } 23.54\%.} \quad \textbf{(Ans.)}$$

Example 12.3. *In a steam turbine steam at 20 bar, 360°C is expanded to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. Assume ideal processes, find per kg of steam the net work and the cycle efficiency.*

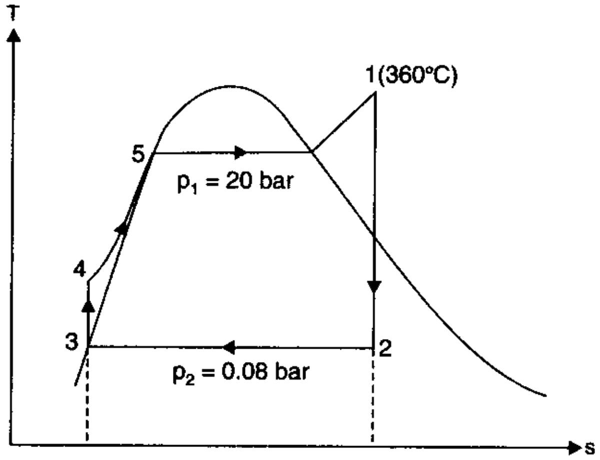


Fig. 12.7

Solution. Boiler pressure, $p_1 = 20$ bar (360°C)

Condenser pressure, $p_2 = 0.08$ bar

From steam tables :

At 20 bar (p_1), 360°C :

$h_1 = 3159.3$ kJ/kg

$s_1 = 6.9917$ kJ/kg-K

At 0.08 bar (p_2) :

$h_3 = h_{f(p_2)} = 173.88$ kJ/kg,

$s_3 = s_{f(p_2)} = 0.5926$ kJ/kg-K

$h_{fg(p_2)} = 2403.1$ kJ/kg, $s_{g(p_2)} = 8.2287$ kJ/kg-K

$v_{f(p_2)} = 0.001008$ m³/kg $\therefore$ $s_{fg(p_2)} = 7.6361$ kJ/kg-K

Now $s_1 = s_2$

$$6.9917 = s_{f(p_2)} + x_2 \, s_{fg(p_2)} = 0.5926 + x_2 \times 7.6361$$

$$\therefore \quad x_2 = \frac{0.69917 - 0.5926}{7.6361} = 0.838$$

$$\therefore \quad h_2 = h_{f(p_2)} + x_2 \, h_{fg(p_2)}$$

$$= 173.88 + 0.838 \times 2403.1 = 2187.68 \text{ kJ/kg.}$$

Net work, W_{net} :

$$W_{net} = W_{turbine} - W_{pump}$$

$$W_{pump} = h_{f_4} - h_{f(p_2)} \; (= h_{f_3}) = v_{f(p_2)} \, (p_1 - p_2)$$

$$= 0.00108 \text{ (m}^3\text{/kg)} \times (20 - 0.08) \times 100 \text{ kN/m}^2$$

$$= 2.008 \text{ kJ/kg}$$

[and $h_{f_4} = 2.008 + h_{f(p_2)} = 2.008 + 173.88 = 175.89$ kJ/kg]

$$W_{turbine} = h_1 - h_2 = 3159.3 - 2187.68 = 971.62 \text{ kJ/kg}$$

$\therefore$ $W_{net} = 971.62 - 2.008 =$ **969.61 kJ/kg. (Ans.)**

Cycle efficiency, η_{cycle} :

$$Q_1 = h_1 - h_{f_4} = 3159.3 - 175.89 = 2983.41 \text{ kJ/kg}$$

$\therefore$ $\eta_{cycle} = \dfrac{W_{net}}{Q_1} = \dfrac{969.61}{2983.41} =$ **0.325 or 32.5%. (Ans.)**

Example 12.4. *A Rankine cycle operates between pressures of 80 bar and 0.1 bar. The maximum cycle temperature is 600°C. If the steam turbine and condensate pump efficiencies are 0.9 and 0.8 respectively, calculate the specific work and thermal efficiency. Relevant steam table extract is given below.*

p(bar)	*t(°C)*	*Specific volume (m³/kg)*		*Specific enthalpy (kJ/kg)*			*Specific entropy (kJ/kg K)*		
		v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
0.1	*45.84*	*0.0010103*	*14.68*	*191.9*	*2392.3*	*2584.2*	*0.6488*	*7.5006*	*8.1494*
80	*295.1*	*0.001385*	*0.0235*	*1317*	*1440.5*	*2757.5*	*3.2073*	*2.5351*	*5.7424*

80 bar, 600°C	*v*	*0.486 m³/kg*
Superheat	*h*	*3642 kJ/kg*
table	*s*	*7.0206 kJ/kgK*

(GATE, 1998)

Solution. Refer Fig. 12.8

At 80 bar, 600°C :

$h_1 = 3642$ kJ / kg ;

$s_1 = 7.0206$ kJ / kg K.

Since $s_1 = s_2$,

$\therefore$ $7.0206 = s_{f_2} + x_2 \, s_{fg_2}$

$= 0.6488 + x_2 \times 7.5006$

or $x_2 = \dfrac{7.0206 - 0.6488}{7.5006} = 0.85$

Now, $h_2 = h_{f_2} + x_2 \, h_{fg_2}$

$= 191.9 + 0.85 \times 2392.3$

$= 2225.36$ kJ/kg

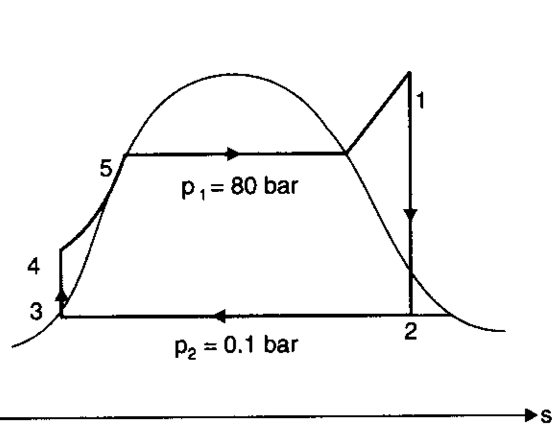


Fig. 12.8

Actual turbine work

$= \eta_{turbine} \times (h_1 - h_2)$

$= 0.9\,(3642 - 2225.36) = 1275$ kJ/kg

Pump work $= v_{f(p_2)}\,(p_1 - p_2)$

$= 0.0010103\,(80 - 0.1) \times \dfrac{10^5}{10^3}$ kN/m² $= 8.072$ kJ/kg

Actual pump work $= \dfrac{8.072}{\eta_{pump}} = \dfrac{8.072}{0.8} = 10.09$ kJ/kg

Specific work $(W_{net}) = 1275 - 10.09 = \mathbf{1264.91\ kJ / kg.}$ **(Ans.)**

Thermal efficiency $= \dfrac{W_{net}}{Q_1}$

where, $Q_1 = h_1 - h_{f_4}$

But $h_{f_4} = h_{f_3}$ + pump work $= 191.9 + 10.09 = 202$ kJ/kg

$\therefore$ Thermal efficiency, $\eta_{th} = \dfrac{1264.91}{3642 - 202} = \mathbf{0.368\ or\ 36.8\ \%}.$ **(Ans.)**

Example 12.5. *A simple Rankine cycle works between pressures 28 bar and 0.06 bar, the initial condition of steam being dry saturated. Calculate the cycle efficiency, work ratio and specific steam consumption.*

Solution.

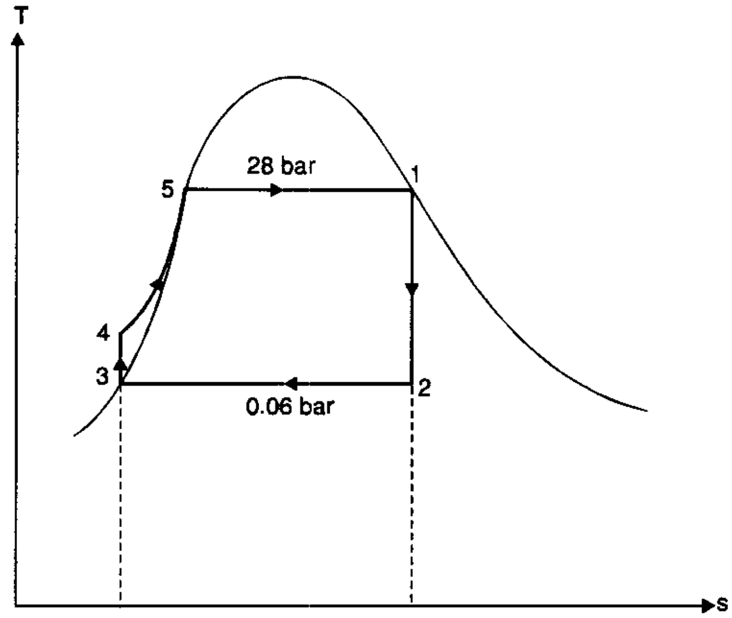


Fig. 12.9

From steam tables,

At 28 bar : $h_1 = 2802$ kJ/kg, $s_1 = 6.2104$ kJ/kg K

At 0.06 bar : $h_{f_2} = h_{f_3} = 151.5$ kJ/kg, $h_{fg_2} = 2415.9$ kJ/kg,

$s_{f_2} = 0.521$ kJ/kg K, $s_{fg_2} = 7.809$ kJ/kg K

$v_f = 0.001\ m^3/kg$

Considering turbine *process 1-2*, we have :

$$s_1 = s_2$$

$$6.2104 = s_{f_2} + x_2\, s_{fg_2} = 0.521 + x_2 \times 7.809$$

$$\therefore \quad x_2 = \frac{6.2104 - 0.521}{7.809} = 0.728$$

$$\therefore \quad h_2 = h_{f_2} + x_2 \, h_{fg_2}$$

$$= 151.5 + 0.728 \times 2415.9 = 1910.27 \text{ kJ/kg}$$

$\therefore$ Turbine work, $W_{turbine} = h_1 - h_2 = 2802 - 1910.27 = 891.73$ kJ/kg

Pump work, $W_{pump} = h_{f_4} - h_{f_3} = v_f (p_1 - p_2)$

$$= \frac{0.001\,(28 - 0.06) \times 10^5}{1000} = 2.79 \text{ kJ/kg}$$

[$\because$ $h_{f_4} = h_{f_3} + 2.79 = 151.5 + 2.79 = 154.29$ kJ/kg]

$\therefore$ Net work, $W_{net} = W_{turbine} - W_{pump}$

$$= 891.73 - 2.79 = 888.94 \text{ kJ/kg}$$

Cycle efficiency $= \dfrac{W_{net}}{Q_1} = \dfrac{888.94}{h_1 - h_{f_4}}$

$$= \frac{888.94}{2802 - 154.29} = \mathbf{0.3357 \text{ or } 33.57\%.} \quad \textbf{(Ans.)}$$

Work ratio $= \dfrac{W_{net}}{W_{turbine}} = \dfrac{888.94}{891.73} = \mathbf{0.997.}$ **(Ans.)**

Specific steam consumption $= \dfrac{3600}{W_{net}} = \dfrac{3600}{888.94} = \mathbf{4.049 \text{ kg/kWh.}}$ **(Ans.)**

☞**Example 12.6.** *In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 35 bar and the exhaust pressure is 0.2 bar. Determine :*

(i) The pump work, *(ii) The turbine work,*
(iii) The Rankine efficiency, *(iv) The condenser heat flow,*
(v) The dryness at the end of expansion.

Assume flow rate of 9.5 kg/s.

Solution. Pressure and condition of steam, at inlet to the turbine,

$p_1 = 35$ bar, $x = 1$

Exhaust pressure, $p_2 = 0.2$ bar

Flow rate, $\dot{m} = 9.5$ kg/s

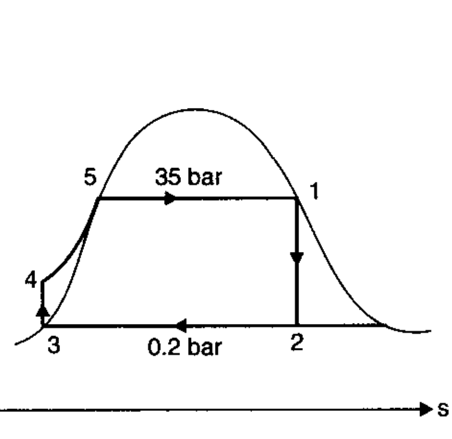


Fig. 12.10

From steam tables :

At 35 bar : $h_1 = h_{g_1} = 2802$ kJ/kg, $s_{g_1} = 6.1228$ kJ/kg K

At 0.26 bar : $h_f = 251.5$ kJ/kg, $h_{fg} = 2358.4$ kJ/kg,

$v_f = 0.001017$ m³/kg, $s_f = 0.8321$ kJ/kg K, $s_{fg} = 7.0773$ kJ/kg K.

(*i*) **The pump work :**

Pump work $= (p_4 - p_3)\, v_f = (35 - 0.2) \times 10^5 \times 0.001017$ J or 3.54 kJ/kg

$$\begin{bmatrix} \text{Also } h_{f_4} - h_{f_3} = \text{Pump work} = 3.54 \\ \therefore \quad h_{f_4} = 251.5 + 3.54 = 255.04 \text{ kJ / kg} \end{bmatrix}$$

Now power required to drive the pump

$= 9.5 \times 3.54$ kJ/s or **33.63 kW. (Ans.)**

(*ii*) **The turbine work :**

$$s_1 = s_2 = s_{f_2} + x_2 \times s_{fg_2}$$

$$6.1228 = 0.8321 + x_2 \times 7.0773$$

$$\therefore \quad x_2 = \frac{6.1228 - 0.8321}{7.0773} = 0.747$$

$$\therefore \quad h_2 = h_{f_2} + x_2\, h_{fg_2} = 251.5 + 0.747 \times 2358.4 = 2013 \text{ kJ/kg}$$

$\therefore$ *Turbine work* $= \dot{m}\,(h_1 - h_2) = 9.5\,(2802 - 2013) =$ **7495.5 kW. (Ans.)**

It may be noted that pump work (33.63 kW) is very small as compared to the turbine work (7495.5 kW).

(*iii*) **The Rankine efficiency :**

$$\eta_{\text{rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_2}} = \frac{2802 - 2013}{2802 - 251.5} = \frac{789}{2550.5} = \mathbf{0.3093 \text{ or } 30.93\%.} \quad \textbf{(Ans.)}$$

(*iv*) **The condenser heat flow :**

The condenser heat flow $= \dot{m}\,(h_2 - h_{f_3}) = 9.5\,(2013 - 251.5) =$ **16734.25 kW. (Ans.)**

(*v*) **The dryness at the end of expansion, x_2 :**

The dryness at the end of expansion,

$x_2 =$ **0.747 or 74.7%. (Ans.)**

Example 12.7. *The adiabatic enthalpy drop across the primemover of the Rankine cycle is 840 kJ/kg. The enthalpy of steam supplied is 2940 kJ/kg. If the back pressure is 0.1 bar, find the specific steam consumption and thermal efficiency.*

Solution. Adiabatic enthalpy drop, $h_1 - h_2 = 840$ kJ/kg

Enthalpy of steam supplied, $h_1 = 2940$ kJ/kg

Back pressure, $p_2 = 0.1$ bar

From steam tables, corresponding to 0.1 bar : $h_f = 191.8$ kJ/kg

Now, $$\eta_{\text{rankine}} = \frac{h_1 - h_2}{h_1 - h_{f_2}} = \frac{840}{2940 - 191.8} = 0.3056 = \mathbf{30.56\%.} \quad \textbf{(Ans.)}$$

Useful work done per kg of steam = 840 kJ/kg

$\therefore$ *Specific steam consumption* $= \frac{1}{840}$ kg/s $= \frac{1}{840} \times 3600 =$ **4.286 kg/kWh. (Ans.)**

☞**Example 12.8.** *A 35 kW (I.P.) system engines consumes 284 kg/h at 15 bar and 250°C. If condenser pressure is 0.14 bar, determine :*

(*i*) *Final condition of steam ;* (*ii*) *Rankine efficiency ;*
(*iii*) *Relative efficiency.*

Solution. Power developed by the engine = 35 kW (I.P.)
Steam consumption = 284 kg/h
Condenser pressure = 0.14 bar
Steam inlet pressure = 15 bar, 250°C.
From steam tables :

At 15 bar, 250°C : h = 2923.3 kJ/kg, s = 6.709 kJ/kg K

At 0.14 bar : h_f = 220 kJ/kg, h_{fg} = 2376.6 kJ/kg,
s_f = 0.737 kJ/kg K, s_{fg} = 7.296 kJ/kg K

(*i*) **Final condition of steam :**

Since steam expands isentropically.

$$\therefore \quad s_1 = s_2 = s_{f_2} + x_2\, s_{fg_2}$$

$$6.709 = 0.737 + x_2 \times 7.296$$

$$\therefore \quad x_2 = \frac{6.709 - 0.737}{7.296} = \mathbf{0.818 \simeq 0.82.} \quad \textbf{(Ans.)}$$

$$\therefore \quad h_2 = h_{f_2} + x_2\, h_{fg_2} = 220 + 0.82 \times 2376.6 = 2168.8 \text{ kJ/kg.}$$

(*ii*) **Rankine efficiency :**

$$\eta_{rankine} = \frac{h_1 - h_2}{h_1 - h_{f_2}} = \frac{2923.3 - 2168.8}{2923.3 - 220} = \mathbf{0.279 \text{ or } 27.9\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Relative efficiency :**

$$\eta_{thermal} = \frac{\text{I.P.}}{\dot{m}\left(h_1 - h_{f_2}\right)} = \frac{35}{\frac{284}{3600}(2923.3 - 220)} = 0.1641 \text{ or } 16.41\%$$

$$\eta_{relative} = \frac{\eta_{thermal}}{\eta_{rankine}} = \frac{0.1641}{0.279}$$

$$= \mathbf{0.588 \text{ or } 58.8\%.} \quad \textbf{(Ans.)}$$

Example 12.9. *Calculate the fuel oil consumption required in a industrial steam plant to generate 5000 kW at the turbine shaft. The calorific value of the fuel is 40000 kJ/kg and the Rankine cycle efficiency is 50%. Assume appropriate values for isentropic turbine efficiency, boiler heat transfer efficiency and combustion efficiency.* **(AMIE Summer, 2000)**

Solution. Power to be generated at the turbine shaft, P = 5000 kW

The calorific value of the fuel, C = 40000 kJ/kg

Rankine cycle efficiency, $\eta_{rankine}$ = 50%

Fuel oil combustion, m_f :

Assume : $\eta_{turbine}$ = 90% ; $\eta_{heat\ transfer}$ = 85% ; $\eta_{combustion}$ = 98%

$$\eta_{rankine} = \frac{\text{Shaft power} / \eta_{turbine}}{m_f \times C \times \eta_{heat\ transfer} \times \eta_{combustion}}$$

or
$$0.5 = \frac{(5000/0.9)}{m_f \times 40000 \times 0.85 \times 0.98}$$

$$\therefore \qquad m_f = \frac{(5000 / 0.9)}{0.5 \times 40000 \times 0.85 \times 0.98} = \mathbf{0.3335\ kg/s\ or\ 1200.6\ kg/h.} \quad \textbf{(Ans.)}$$

12.3. MODIFIED RANKINE CYCLE

Figures 12.11 and 12.12 show the modified Rankine cycle on *p-V* and *T-s* diagrams (neglecting pump work) respectively. It will be noted that *p-V* diagram is very narrow at the toe *i.e.,* point '2'' and the work obtained near to *e* is very small. In fact this work is too inadequate to overcome friction (due to reciprocating parts) even. Therefore, the adiabatic is terminated at '2' ; the pressure drop decreases suddenly whilst the volume remains constant. This operation is represented by the line 2-3. By this doing the stroke length is reduced ; in other words the cylinder dimensions reduce but at the expense of small loss of work (area 2-3-2') which, however, is negligibly small.

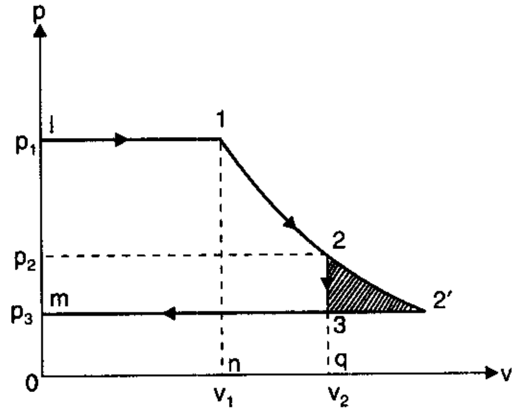


Fig. 12.11. *p-V* diagram of Modified Rankine Cycle.

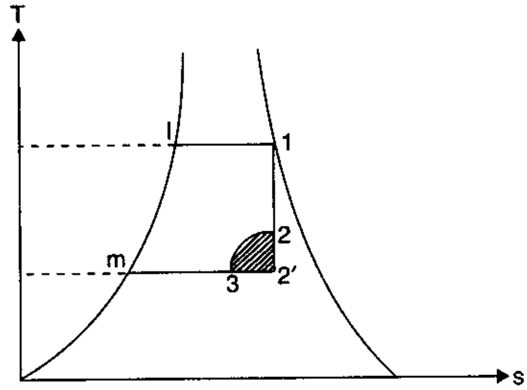


Fig. 12.12. *T-s* diagram of Modified Rankine cycle.

The work done during the modified Rankine cycle can be calculated in the following way :

Let p_1, v_1, u_1 and h_1 correspond to initial condition of steam at '1'.

p_2, v_2, u_2 and h_2 correspond to condition of steam at '2'.

p_3, h_3 correspond to condition of steam at '3''.

Work done during the cycle/kg of steam

= area *l*-1-2-3-*m*

= area '*o*-*l*-1-*n*' + area '1-2-*q*-*n*' – area '*o*-*m*-3-*q*'

$= p_1v_1 + (u_1 - u_2) - p_3v_2$

Heat supplied $= h_1 - h_{f_3}$

∴ The modified Rankine efficiency

$$= \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{p_1v_1 + (u_1 - u_2) - p_3v_2}{h_1 - h_{f_3}} \quad ...(12.6)$$

Alternative method for finding modified Rankine efficiency :

Work done during the cycle/kg of steam

= area '*l*-1-2-3-*m*'

= area '*l*-1-2-*s*' + area '*s*-2-3-*m*'

$= (h_1 - h_2) + (p_2 - p_3)\, v_2$

Heat supplied $= h_1 - h_{f_3}$

Modified Rankine efficiency $= \dfrac{\text{Work done}}{\text{Heat supplied}}$

$$= \frac{(h_1 - h_2) + (p_2 - p_3)\, v_2}{h_1 - h_{f_3}} \quad ...(12.7)$$

Note. Modified Rankine cycle is used for 'reciprocating steam engines' because stroke length and hence cylinder size is reduced with the sacrifice of practically a quite negligible amount of work done.

☞**Example 12.10. (Modified Rankine Cycle).** *Steam at a pressure of 15 bar and 300°C is delivered to the throttle of an engine. The steam expands to 2 bar when release occurs. The steam exhaust takes place at 1.1 bar. A performance test gave the result of the specific steam consumption of 12.8 kg/kWh and a mechanical efficiency of 80 per cent. Determine :*

(*i*) *Ideal work or the modified Rankine engine work per kg.*

(*ii*) *Efficiency of the modified Rankine engine or ideal thermal efficiency.*

(*iii*) *The indicated and brake work per kg.*

(*iv*) *The brake thermal efficiency.*

(*v*) *The relative efficiency on the basis of indicated work and brake work.*

Solution. Fig. 12.13 shows the *p-v* and *T-s* diagrams for modified Rankine cycle.

From steam tables :

1. ***At 15 bar, 300°C :*** $h_1 = 3037.6$ kJ/kg, $v_1 = 0.169$ m³/kg, $s_1 = 6.918$ kJ/kg K.

2. ***At 2 bar :*** $t_{s_2} = 120.2$°C, $h_{f_2} = 504.7$ kJ/kg, $h_{fg_2} = 2201.6$ kJ/kg, $s_{f_2} = 1.5301$ kJ/kg K, $s_{fg_2} = 5.5967$ kJ/kg K, $v_{f_2} = 0.00106$ m³/kg, $v_{g_2} = 0.885$ m³/kg.

3. ***At 1.1 bar :*** $t_{s_3} = 102.3$°C, $h_{f_3} = 428.8$ kJ/kg, $h_{fg_3} = 2250.8$ kJ/kg, $s_{f_3} = 1.333$ kJ/kg K, $s_{fg_3} = 5.9947$ kJ/kg K, $v_{f_3} = 0.001$ m³/kg, $v_{g_3} = 1.549$ m³/kg.

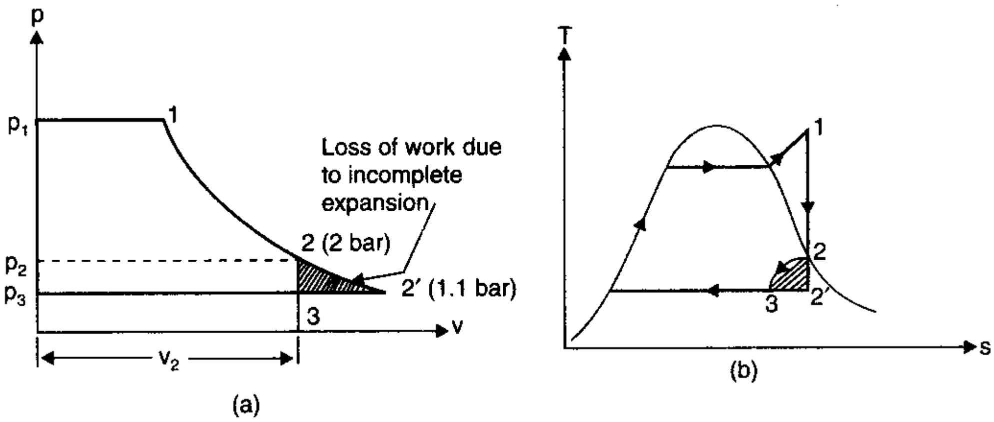


Fig. 12.13. *p-V* and *T-s* diagrams.

During isentropic expansion 1-2, we have

$$s_1 = s_2$$

$$6.918 = s_{f_2} + x_2\, s_{fg_2} = 1.5301 + x_2 \times 5.5967$$

$$\therefore \qquad x_2 = \frac{6.918 - 1.5301}{5.5967} = 0.96.$$

Then $\qquad h_2 = h_{f_2} + x_2 h_{fg_2} = 504.7 + 0.96 \times 2201.6 = 2618.2$ kJ/kg

$$v_2 = x_2\, v_{g_2} + (1 - x_2)\, v_{f_2}$$
$$= 0.96 \times 0.885 + (1 - 0.96) \times 0.00106 = 0.849 \text{ m}^3/\text{kg}.$$

(*i*) **Ideal work :**

Ideal work or modified Rankine engine work/kg,

$$W = (h_1 - h_2) + (p_2 - p_3)\, v_2$$
$$= (3037.6 - 2618.2) + (2 - 1.1) \times 10^5 \times 0.849/1000$$
$$= 419.4 + 76.41 = \mathbf{495.8\ kJ/kg.}\quad \textbf{(Ans.)}$$

(*ii*) **Rankine engine efficiency :**

$$\eta_{\text{rankine}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{495.8}{(h_1 - h_{f_3})}$$

$$= \frac{495.8}{3037.6 - 428.8} = \mathbf{0.19 \text{ or } 19\%.}\quad \textbf{(Ans.)}$$

(*iii*) **Indicated and brake work per kg :**

Indicated work/kg, $\qquad W_{\text{indicated}} = \frac{\text{I.P.}}{\dot{m}}$

$$= \frac{1 \times 3600}{12.8} = \mathbf{281.25\ kJ/kg.}\quad \textbf{(Ans.)}$$

Brake work/kg, $\qquad W_{\text{brake}} = \frac{\text{B.P.}}{\dot{m}} = \frac{\eta_{\text{mech.}} \times \text{I.P.}}{\dot{m}}$

$$= \frac{0.8 \times 1 \times 3600}{12.8} = \mathbf{225\ kJ/kg.}\quad \textbf{(Ans.)}$$

(*iv*) **Brake thermal efficiency :**

$$\text{Brake thermal efficiency} = \frac{W_{\text{brake}}}{h_1 - h_{f_3}} = \frac{225}{3037.6 - 428.8} = \textbf{0.086 or 8.6\%.} \quad \textbf{(Ans.)}$$

(*v*) **Relative efficiency :**

Relative efficiency *on the basis of indicated work*

$$= \frac{\dfrac{W_{\text{indicated}}}{h_1 - h_{f_3}}}{\dfrac{W}{h_1 - h_{f_3}}} = \frac{W_{\text{indicated}}}{W} = \frac{281.25}{495.8} = \textbf{0.567 or 56.7\%.} \quad \textbf{(Ans.)}$$

Relative efficiency *on the basis of brake work*

$$= \frac{\dfrac{W_{\text{indicated}}}{(h_1 - h_{f_3})}}{\dfrac{W}{(h_1 - h_{f_3})}} = \frac{W_{\text{brake}}}{W} = \frac{225}{495.8} = \textbf{0.4538 or 45.38\%.} \quad \textbf{(Ans.)}$$

Example 12.11. *Superheated steam at a pressure of 10 bar and 400°C is supplied to a steam engine. Adiabatic expansion takes place to release point at 0.9 bar and it exhausts into a condenser at 0.3 bar. Neglecting clearance determine for a steam flow rate of 1.5 kg/s :*

(*i*) *Quality of steam at the end of expansion and the end of constant volume operation.*

(*ii*) *Power developed.*

(*iii*) *Specific steam consumption.*

(*iv*) *Modified Rankine cycle efficiency.*

Solution. Fig. 12.14 shows the *p-V* and *T-s* diagrams for modified Rankine cycle.

From steam tables :

1. At 10 bar, 400°C : $h_1 = 3263.9$ kJ/kg, $v_1 = 0.307$ m³/kg, $s_1 = 7.465$ kJ/kg K

2. At 0.9 bar : $t_{s_2} = 96.7$°C, $h_{g_2} = 2670.9$ kJ/kg, $s_{g_2} = 7.3954$ kJ/kg K,

$v_{g_2} = 1.869$ m³/kg

3. At 0.3 bar : $h_{f_3} = 289.3$ kJ/kg, $v_{g_3} = 5.229$ m³/kg

(*i*) **Quality of steam at the end of expansion,** T_{sup2} **:**

For isentropic expansion 1-2, we have

$$s_1 = s_2$$

$$= s_{g_2} + c_p \log_e \frac{T_{sup\,2}}{T_{s_2}}$$

$$7.465 = 7.3954 + 2.1 \log_e \frac{T_{sup2}}{(96.7 + 273)}$$

$$\left(\frac{7.465 - 7.3954}{2.1}\right) = \log_e \frac{T_{sup2}}{369.7} \quad \text{or} \quad \log_e \frac{T_{sup2}}{369.7} = 0.0033$$

$$\frac{T_{sup2}}{369.7} = 1.0337 \quad \text{or} \quad T_{sup2} = 382 \text{ K}$$

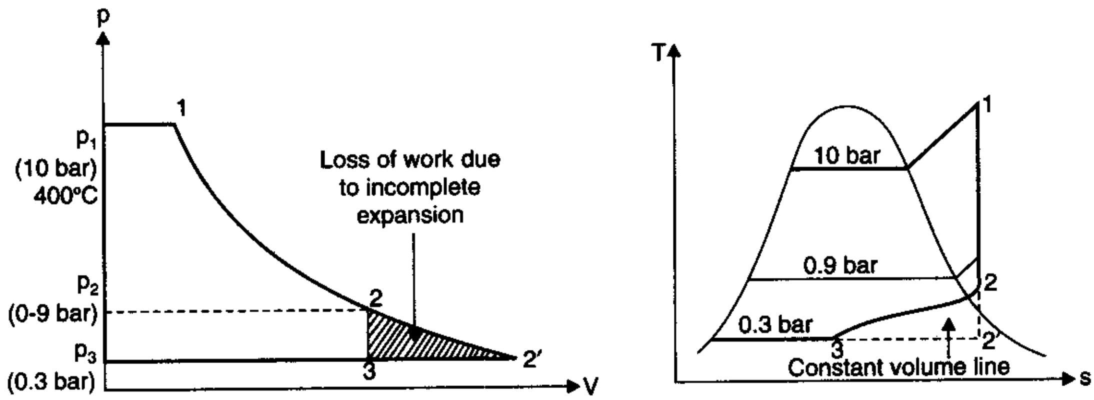


Fig. 12.14. *p-V* and *T-s* diagrams.

or $$t_{sup2} = 382 - 273 = \mathbf{109°C.}\quad \textbf{(Ans.)}$$

$$\therefore \quad h_2 = h_{g_2} + c_{ps}(T_{sup2} - T_{s_2})$$

$$= 2670.9 + 2.1\,(382 - 366.5) = 2703.4 \text{ kJ/kg.}$$

(*ii*) **Quality of steam at the end of constant volume operation, x_3 :**

For calculating v_2 using the relation

$$\frac{v_{g_2}}{T_{s_2}} = \frac{v_2}{T_{sup2}} \quad \text{(Approximately)}$$

$$\frac{1.869}{369.7} = \frac{v_2}{382}$$

or $$v_2 = \frac{1.869 \times 382}{369.7} = 1.931 \text{ m}^3\text{/kg}$$

Also $$v_2 = v_3 = x_3\, v_{g_3}$$

$$1.931 = x_3 \times 5.229$$

or $$\mathbf{x_3} = \frac{1.931}{5.229} = \mathbf{0.37.} \quad \textbf{(Ans.)}$$

(*iii*) **Power developed, P :**

Work done $$= (h_1 - h_2) + (p_2 - p_3)\, v_2$$

$$= (3263.9 - 2703.4) + \frac{(0.75 - 0.3) \times 10^5 \times 1.931}{1000}$$

$$= 560.5 + 86.9 = 647.4 \text{ kJ/kg}$$

∴ **Power developed** = Steam flow rate × work done (per kg)

$$= 1 \times 647.4 = \mathbf{647.4\ kW.} \quad \textbf{(Ans.)}$$

(*iv*) **Specific steam consumption, ssc :**

$$\textbf{ssc} = \frac{3600}{\text{Power}} = \frac{1 \times 3600}{647.4} = \mathbf{5.56\ kg/kWh.} \quad \textbf{(Ans.)}$$

(*v*) **Modified Rankine cycle efficiency, η_{mR} :**

$$\eta_{mR} = \frac{(h_1 - h_2) + (p_2 - p_3)\, v_2}{h_1 - h_{f_3}}$$

$$= \frac{647.4}{3263.9 - 289.3} = \mathbf{0.217 \text{ or } 21.7\%.} \quad \textbf{(Ans.)}$$

12.4. REGENERATIVE CYCLE

In the Rankine cycle it is observed that the condensate which is fairly at low temperature has an irreversible mixing with hot boiler water and this results in decrease of cycle efficiency. Methods are, therefore, adopted to heat the feed water from the hot well of condenser irreversibly by interchange of heat within the system and thus improving the cycle efficiency. This heating method is called regenerative feed heat and the cycle is called *regenerative cycle*.

The principle of regeneration can be practically utilised by extracting steam from the turbine at several locations and supplying it to the regenerative heaters. The resulting cycle is known as *regenerative or bleeding cycle.* The heating arrangement comprises of : (*i*) For medium capacity turbines—not more than 3 heaters ; (*ii*) For high pressure high capacity turbines—not more than 5 to 7 heaters ; and (*iii*) For turbines of super critical parameters 8 to 9 heaters. The most advantageous condensate heating temperature is selected depending on the turbine throttle conditions and this determines the number of heaters to be used. The final condensate heating temperature is kept 50 to 60°C below the boiler saturated steam temperature so as to prevent evaporation of water in the feed mains following a drop in the boiler drum pressure. The conditions of steam bled for each heater are so selected that the temperature of saturated steam will be 4 to 10°C higher than the final condensate temperature.

Fig. 12.15 (*a*) shows a diagrammatic layout of a condensing steam power plant in which a surface condenser is used to condense all the steam that is not extracted for feed water heating. The turbine is double extracting and the boiler is equipped with a superheater. The cycle diagram (*T-s*) would appear as shown in Fig. 12.15 (*b*). This arrangement constitutes a *regenerative cycle.*

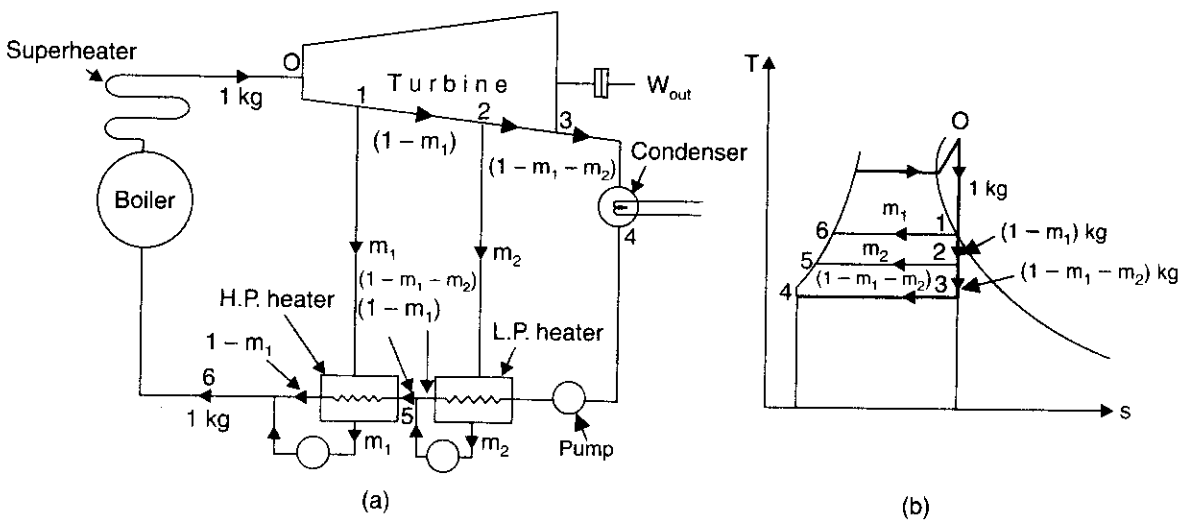


Fig. 12.15. Regenerative cycle.

Let, m_1 = kg of high pressure (H.P.) steam per kg of steam flow,

m_2 = kg of low pressure (L.P.) steam extracted per kg of steam flow, and

$(1 - m_2 - m_2)$ = kg of steam entering condenser per kg of steam flow.

Energy/Heat balance equation for H.P. heater :

$$m_1 (h_1 - h_{f_6}) = (1 - m_1)(h_{f_6} - h_{f_5})$$

or $$m_1[(h_1 - h_{f_6}) + (h_{f_6} - h_{f_5})] = (h_{f_6} - h_{f_5})$$

or $$m_1 = \frac{h_{f_6} - h_{f_5}}{h_1 - h_{f_5}} \quad ...(12.8)$$

Energy/Heat balance equation for L.P. heater :

$$m_2 (h_2 - h_{f_5}) = (1 - m_1 - m_2)(h_{f_5} - h_{f_3})$$

or $$m_2[(h_2 - h_{f_5}) + (h_{f_5} - h_{f_3})] = (1 - m_1)(h_{f_5} - h_{f_3})$$

or $$m_2 = \frac{(1 - m_1)(h_{f_5} - h_{f_3})}{(h_2 - h_{f_3})} \quad ...(12.9)$$

All enthalpies may be determined ; therefore m_1 and m_2 may be found. The maximum temperature to which the water can be heated is dictated by that of bled steam. The condensate from the bled steam is added to feed water.

Neglecting pump work :

The heat supplied externally in the cycle

$$= (h_0 - h_{f_6})$$

Isentropic work done $= m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)$

The thermal efficiency of regenerative cycle is

$$\eta_{\text{thermal}} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2)(h_0 - h_3)}{(h_0 - h_{f_6})} \quad ...(12.10)$$

[The work done by the turbine may also be calculated by summing up the products of the steam flow and the corresponding heat drop in the turbine stages.

i.e., Work done $= (h_0 - h_1) + (1 - m_1)(h_1 - h_2) + (1 - m_1 - m_2)(h_2 - h_3)$]

Advantages of Regenerative cycle over Simple Rankine cycle :

1. The heating process in the boiler tends to become reversible.

2. The thermal stresses set up in the boiler are minimised. This is due to the fact that temperature ranges in the boiler are reduced.

3. The thermal efficiency is improved because the average temperature of heat addition to the cycle is increased.

4. Heat rate is reduced.

5. The blade height is less due to the reduced amount of steam passed through the low pressure stages.

6. Due to many extractions there is an improvement in the turbine drainage and it reduces erosion due to moisture.

7. A small size condenser is required.

Disadvantages :

1. The plant becomes more complicated.
2. Because of addition of heaters greater maintenance is required.
3. For given power a large capacity boiler is required.
4. The heaters are costly and the gain in thermal efficiency is not much in comparison to the heavier costs.

Note. In the absence of precise information (regarding actual temperature of the feed water entering and leaving the heaters and of the condensate temperatures) the following assumption should always be made while doing calculations :

1. Each heater is ideal and bled steam just condenses.
2. The feed water is heated to saturation temperature at the pressure of bled steam.
3. Unless otherwise stated the work done by the pumps in the system is considered negligible.
4. There is equal temperature rise in all the heaters (usually 10°C to 15°C).

Example 12.12. *A steam turbine is fed with steam having an enthalpy of 3100 kJ/kg. It moves out of the turbine with an enthalpy of 2100 kJ/kg. Feed heating is done at a pressure of 3.2 bar with steam enthalpy of 2500 kJ/kg. The condensate from a condenser with an enthalpy of 125 kJ/kg enters into the feed heater. The quantity of bled steam is 11200 kg/h. Find the power developed by the turbine. Assume that the water leaving the feed heater is saturated liquid at 3.2 bar and the heater is direct mixing type. Neglect pump work.*

Solution. Arrangement of the components is shown in Fig. 12.16.

At 3.2 bar, $h_{f_2} = 570.9$ kJ/kg.

Consider m kg out of 1 kg is taken to the feed heater (Fig. 12.16).

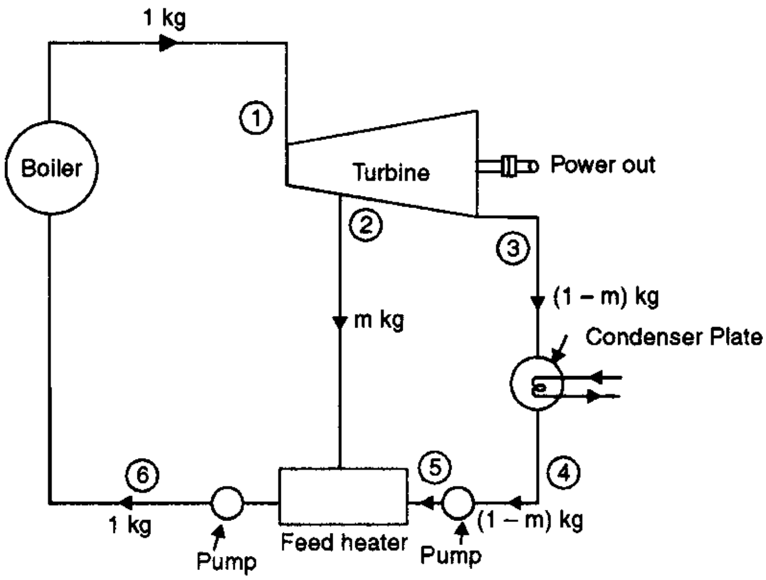


Fig. 12.16

Energy balance for the feed heater is written as :

$$m h_2 + (1 - m)\, h_{f_5} = 1 \times h_{f_2}$$

$$m \times 2100 + (1 - m) \times 125 = 1 \times 570.9$$

$$2100\, m + 125 - 125\, m = 570.9$$

$$1975\, m = 570.9 - 125$$

$\therefore$ $m = 0.226$ kg per kg of steam supplied to the turbine

$\therefore$ Steam supplied to the turbine per hour

$$= \frac{11200}{0.226} = 49557.5 \text{ kg/h}$$

Net work developed per kg of steam

$$= (h_1 - h_2) + (1 - m)(h_2 - h_3)$$

$$= (3100 - 2500) + (1 - 0.226)(2500 - 2100)$$

$$= 600 + 309.6 = 909.6 \text{ kJ/kg}$$

$\therefore$ **Power developed by the turbine**

$$= 909.6 \times \frac{49557.5}{3600} \text{ kJ/s}$$

$= \mathbf{12521.5\ kW.}$ **(Ans.)** $(\because 1 \text{ kJ/s} = 1 \text{ kW})$

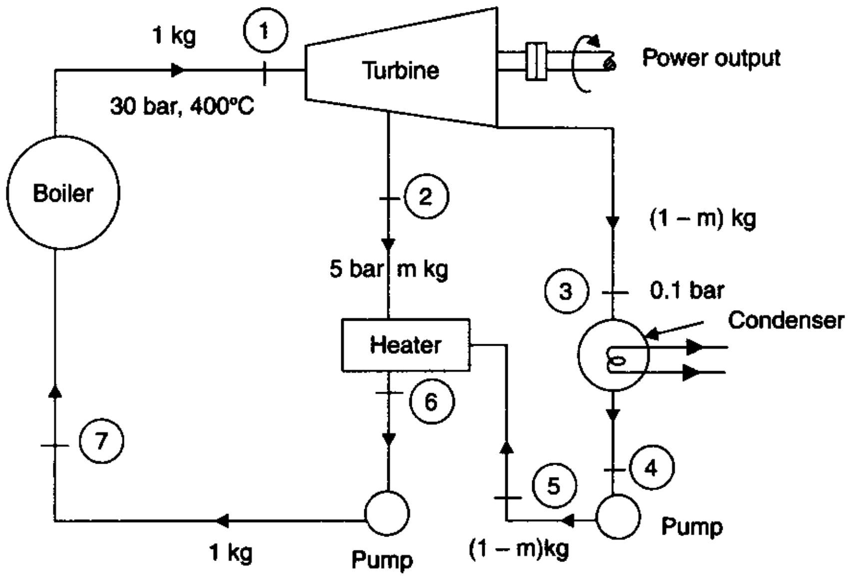
☞**Example 12.13.** *In a single-heater regenerative cycle the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.10 bar. The feed water heater is a direct contact type which operates at 5 bar. Find :*

(i) The efficiency and the steam rate of the cycle.

(ii) The increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration).

Pump work may be neglected.

Solution. Fig. 12.17 shows the flow, T-s and h-s diagrams.



(a)

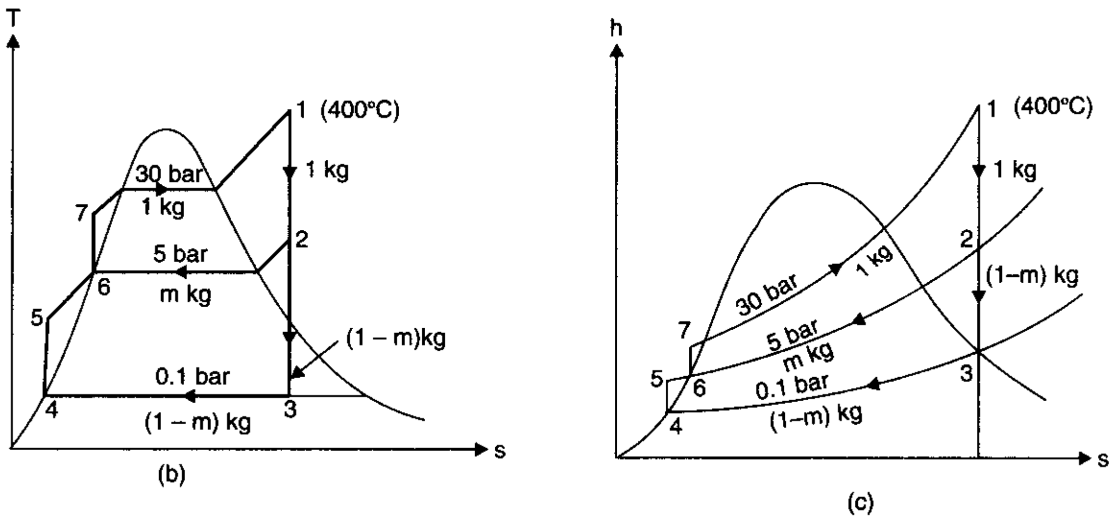


Fig. 12.17

From steam tables :

At 30 bar, 400°C : $h_1 = 3230.9$ kJ/kg, $s_1 = 6.921$ kJ/kg K $= s_2 = s_3$,

At 5 bar : $s_f = 1.8604$, $s_g = 6.8192$ kJ/kg K, $h_f = 640.1$ kJ/kg

Since $s_2 > s_g$, the state 2 must lie in the superheated region. From the table for superheated steam $t_2 = 172$°C, $h_2 = 2796$ kJ/kg.

At 0.1 bar : $s_f = 0.649$, $s_{f_g} = 7.501$, $h_f = 191.8$, $h_{f_g} = 2392.8$

Now, $s_2 = s_3$

i.e., $6.921 = s_{f_3} + x_3\, s_{fg_3} = 0.649 + x_3 \times 7.501$

$$\therefore \quad x_3 = \frac{6.921 - 0.649}{7.501} = 0.836$$

$$\therefore \quad h_3 = h_{f_3} + x_3\, h_{fg_3} = 191.8 + 0.836 \times 2392.8 = 2192.2 \text{ kJ/kg}$$

Since pump work is neglected

$$h_{f_4} = 191.8 \text{ kJ/kg} = h_{f_5}$$

$$h_{f_6} = 640.1 \text{ kJ/kg (at 5 bar)} = h_{f_7}$$

Energy balance for heater gives

$$m\,(h_2 - h_{f_6}) = (1 - m)\,(h_{f_6} - h_{f_5})$$

$$m\,(2796 - 640.1) = (1 - m)\,(640.1 - 191.8) = 448.3\,(1 - m)$$

$$2155.9\, m = 448.3 - 448.3\, m$$

$$\therefore \quad m = 0.172 \text{ kg}$$

$\therefore$ Turbine work,

$$W_T = (h_1 - h_2) + (1 - m)\,(h_2 - h_3)$$
$$= (3230.9 - 2796) + (1 - 0.172)\,(2796 - 2192.2)$$
$$= 434.9 + 499.9 = 934.8 \text{ kJ/kg}$$

Heat supplied, $Q_1 = h_1 - h_{f_6} = 3230.9 - 640.1 = 2590.8$ kJ/kg.

(*i*) **Efficiency of cycle, η_{cycle} :**

$$\eta_{cycle} = \frac{W_T}{Q_1} = \frac{934.8}{2590.8} = \mathbf{0.3608 \text{ or } 36.08\%.} \quad \textbf{(Ans.)}$$

$$\textbf{Steam rate} = \frac{3600}{934.8} = \mathbf{3.85 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

(*ii*) $$T_{m_1} = \frac{h_1 - h_{f_7}}{s_1 - s_7} = \frac{2590.8}{6.921 - 1.8604} = 511.9 \text{ K} = 238.9°\text{C}.$$

T_{m_1} (without regeneration)

$$= \frac{h_1 - h_{f_4}}{s_1 - s_4} = \frac{3230.9 - 191.8}{6.921 - 0.649} = \frac{3039.1}{6.272} = 484.5 \text{ K} = 211.5°\text{C}.$$

Increase in T_{m_1} due to regeneration

$= 238.9 - 211.5 =$ **27.4°C. (Ans.)**

W_T (without regeneration)

$= h_1 - h_3 = 3230.9 - 2192.2 = 1038.7$ kJ/kg

Steam rate without regeneration

$$= \frac{3600}{1038.7} = 3.46 \text{ kg/kWh}$$

$\therefore$ **Increase in steam rate due to regeneration**

$= 3.85 - 3.46 =$ **0.39 kg/kWh. (Ans.)**

$$\eta_{cycle} \text{ (without regeneration)} = \frac{h_1 - h_3}{h_1 - h_{f_4}} = \frac{1038.7}{3230.9 - 191.8} = \mathbf{0.3418 \text{ or } 34.18\%.} \quad \textbf{(Ans.)}$$

Increase in cycle efficiency due to regeneration

$= 36.08 - 34.18 =$ **1.9%. (Ans.)**

Example 12.14. *Steam is supplied to a turbine at a pressure of 30 bar and a temperature of 400°C and is expanded adiabatically to a pressure of 0.04 bar. At a stage of turbine where the pressure is 3 bar a connection is made to a surface heater in which the feed water is heated by bled steam to a temperature of 130°C. The condensed steam from the feed heater is cooled in a drain cooler to 27°C. The feed water passes through the drain cooler before entering the feed heater. The cooled drain water combines with the condensate in the well of the condenser.*

Assuming no heat losses in the steam, calculate the following :

(i) Mass of steam used for feed heating per kg of steam entering the turbine ;

(ii) Thermal efficiency of the cycle.

Solution. Refer Fig. 12.18.

From steam tables :

At 3 bar : $t_s = 133.5°\text{C}$, $h_f = 561.4$ kJ/kg.

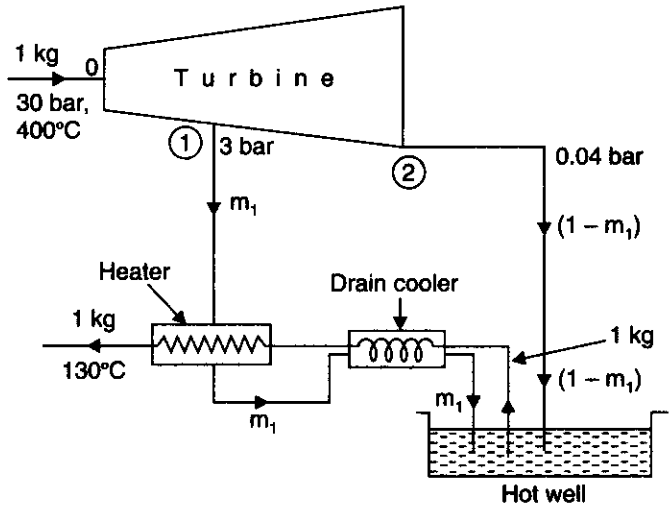
At 0.04 bar : $t_s = 29°\text{C}$, $h_f = 121.5$ kJ/kg.

From Mollier chart :

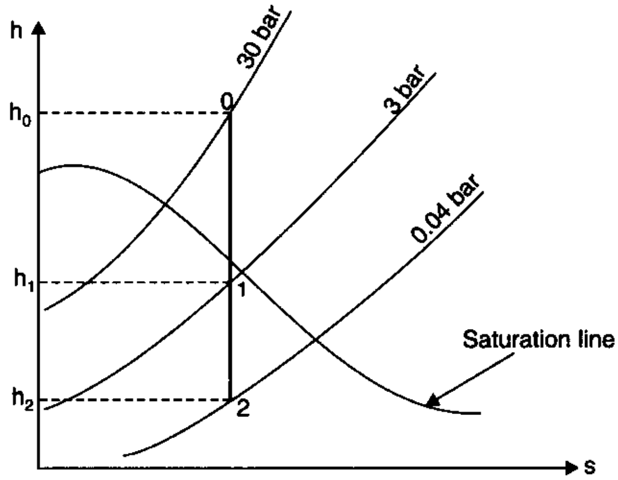
$h_0 = 3231$ kJ/kg (at 30 bar, 400°C)

$h_1 = 2700$ kJ/kg (at 3 bar)

$h_2 = 2085$ kJ/kg (at 0.04 bar).



(a)



(b)

Fig. 12.18

(*i*) **Mass of steam used, m_1 :**

Heat lost by the steam = Heat gained by water.

Taking the feed-heater and drain-cooler combined, we have :

$$m_1 (h_1 - h_{f_2}) = 1 \times 4.186 \, (130 - 27)$$

or $$m_1 (2700 - 121.5) = 4.186 \, (130 - 27)$$

$\therefore$ $$\mathbf{m_1} = \frac{4.186 \, (130 - 27)}{(2700 - 121.5)} = \mathbf{0.1672 \; kg.} \quad \textbf{(Ans.)}$$

(*ii*) **Thermal efficiency of the cycle :**

Work done per kg of steam

$$= 1(h_0 - h_1) + (1 - m_1)(h_1 - h_2)$$

$$= 1(3231 - 2700) + (1 - 0.1672)(2700 - 2085)$$
$$= 1043.17 \text{ kJ/kg}$$

Heat supplied per kg of steam $= h_0 - 1 \times 4.186 \times 130$

$$= 3231 - 544.18 = 2686.82 \text{ kJ/kg.}$$

$$\eta_{\text{Thermal}} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{1043.17}{2686.82} = \mathbf{0.3882 \text{ or } 38.82\%}. \quad \textbf{(Ans.)}$$

Example 12.15. *Steam is supplied to a turbine at 30 bar and 350°C. The turbine exhaust pressure is 0.08 bar. The main condensate is heated regeneratively in two stages by steam bled from the turbine at 5 bar and 1.0 bar respectively. Calculate masses of steam bled off at each pressure per kg of steam entering the turbine and the theoretical thermal efficiency of the cycle.*

Solution. Refer Fig. 12.19.

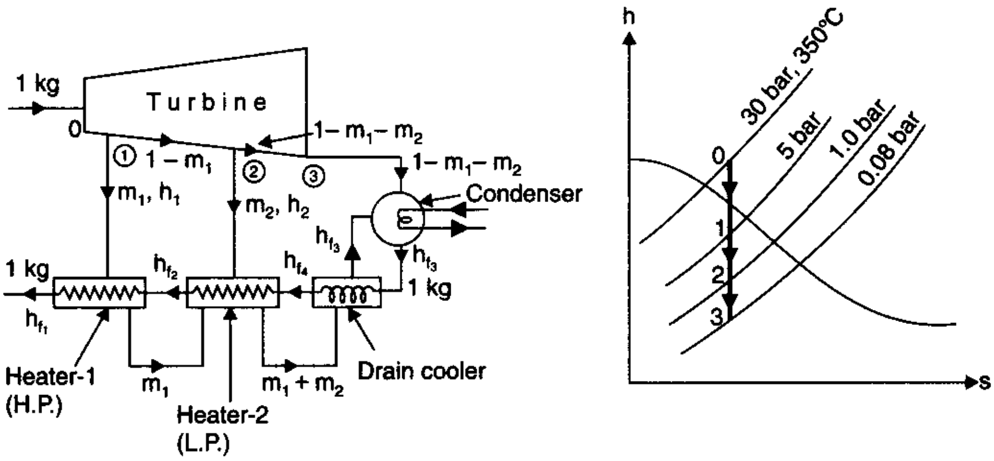


Fig. 12.19

The following *assumptions* are made :

1. The condensate is heated to the saturation temperature in each heater.

2. The drain water from H.P. (high pressure) heater passes into the steam space of the L.P. (low pressure) heater without loss of heat.

3. The combined drains from the L.P. heater are cooled in a drain cooler to the condenser temperature.

4. The expansion of the steam in the turbine is adiabatic and frictionless.

Enthalpy at 30 bar, 350°C, h_0 = 3115.3 kJ/kg.

After adiabatic expansion (from Mollier chart)

Enthalpy at 5 bar, h_1 = 2720 kJ/kg

Enthalpy at 1.0 bar, h_2 = 2450 kJ/kg

Enthalpy at 0.08 bar, h_3 = 2120 kJ/kg

From steam tables : h_{f_1} = 640.1 kJ/kg (at 5.0 bar)

h_{f_2} = 417.5 kJ/kg (at 1.0 bar)

h_{f_3} = 173.9 kJ/kg (at 0.08 bar)

At heater No. 1 :

$$m_1h_1 + h_{f_2} = m_1h_{f_1} + h_{f_1}$$

$$\mathbf{m_1} = \frac{h_{f_1} - h_{f_2}}{h_1 - h_{f_1}} = \frac{640.1 - 417.5}{2720 - 640.1} = \mathbf{0.107\ kJ/kg\ of\ entering\ steam.}$$

(Ans.)

At heater No. 2 :

$$m_2h_2 + m_1h_{f_1} + h_{f_4} = (m_1 + m_2)\ h_{f_2} + h_{f_2} \quad ...(i)$$

At drain cooler :

$$(m_1 + m_2)\ h_{f_2} + h_{f_3} = h_{f_4} + (m_1 + m_2)\ h_{f_3}$$

$$\therefore \quad h_{f_4} = (m_1 + m_2)(h_{f_2} - h_{f_3}) + h_{f_3} \quad ...(ii)$$

Inserting the value of h_{f_4} in eqn. (*i*), we get

$$m_2h_2 + m_1h_{f_1} + (m_1 + m_2)(h_{f_2} - h_{f_3}) + h_{f_3} = (m_1 + m_2)\ h_{f_2} + h_{f_2}$$

$$m_2h_2 + m_1h_{f_1} + (m_1 + m_2)h_{f_2} - (m_1 + m_2)h_{f_3} + h_{f_3} = (m_1 + m_2)h_{f_2} + h_{f_2}$$

$$m_2h_2 + m_1h_{f_1} - m_1h_{f_3} - m_2h_{f_3} + h_{f_3} = h_{f_2}$$

$$m_2(h_2 - h_{f_3}) = (h_{f_2} - h_{f_3}) - m_1(h_{f_1} - h_{f_3})$$

$$\mathbf{m_2} = \frac{(h_{f_2} - h_{f_3}) - m_1(h_{f_1} - h_{f_3})}{(h_2 - h_{f_3})}$$

$$= \frac{(417.5 - 173.9) - 0.107(640.1 - 173.9)}{(2450 - 173.9)}$$

$$= \frac{193.7}{2276.1} = \mathbf{0.085\ kJ/kg.\ (Ans.)}$$

Work done $= 1\ (h_0 - h_1) + (1 - m_1)(h_1 - h_2) + (1 - m_1 - m_2)(h_2 - h_3)$

$= 1\ (3115.3 - 2720) + (1 - 0.107)(2720 - 2450) + (1 - 0.107 - 0.085)(2450 - 2120)$

$= 395.3 + 241.11 + 266.64 = 903.05$ kJ/kg

Heat supplied/kg $= h_0 - h_{f_1}$

$= 3115.3 - 640.1 = 2475.2$ kJ/kg

$\therefore$ **Thermal efficiency of the cycle**

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{903.05}{2475.2} = \mathbf{0.3648\ or\ 36.48\%.\ (Ans.)}$$

☞**Example 12.16.** *Steam at a pressure of 20 bar and 250°C enters a turbine and leaves it finally at a pressure of 0.05 bar. Steam is bled off at pressures of 5.0, 1.5 and 0.3 bar. Assuming (i) that the condensate is heated in each heater upto the saturation temperature of the steam in that heater, (ii) that the drain water from each heater is cascaded through a trap into the next heater on the low pressure side of it, (iii) that the combined drains from the heater operating at 0.3 bar are cooled in a drain cooler to condenser temperature, calculate the following :*

(*i*) *Mass of bled steam for each heater per kg of steam entering the turbine*

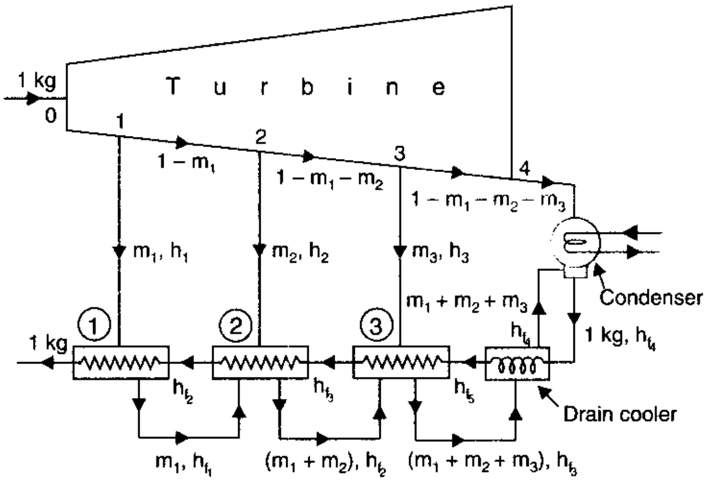
(*ii*) *Thermal efficiency of the cycle,* (*iii*) *Thermal efficiency of the Rankine cycle*

(*iv*) *Theoretical gain due to regenerative feed heating,*

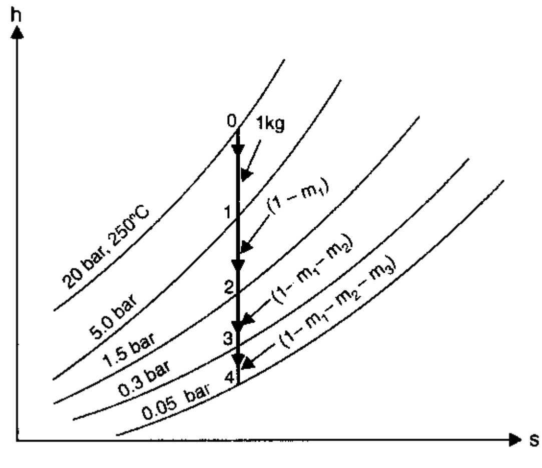
(*v*) *Steam consumption in kg/kWh with or without regenerative feed heating, and*

(*vi*) *Quantity of steam passing through the last stage nozzle of a 50000 kW turbine with and without regenerative feed heating.*

Solution. Refer Fig. 12.20 (*a*), (*b*).



(a)



(b)

Fig. 12.20

From Mallier Chart : h_0 = 2905 kJ/kg, h_1 = 2600 kJ/kg, h_2 = 2430 kJ/kg

h_3 = 2210 kJ/kg, h_4 = 2000 kJ/kg

From steam tables :

At 5 bar : h_{f_1} = 640.1 kJ/kg

At 1.5 bar : h_{f_2} = 467.1 kJ/kg

At 0.3 bar : $h_{f_3} = 289.3$ kJ/kg

At 0.05 bar : $h_{f_4} = 137.8$ kJ/kg.

(*i*) **Mass of bled steam for each heater per kg of steam :**

Using heat balance equation :

At heater No. 1 :

$$m_1h_1 + h_{f_2} = m_1h_{f_1} + h_{f_1}$$

$$\therefore \quad \mathbf{m_1} = \frac{h_{f_1} - h_{f_2}}{h_1 - h_{f_1}} = \frac{640.1 - 467.1}{2600 - 640.1}$$

= **0.088 kJ/kg of entering steam. (Ans.)**

At heater No. 2 :

$$m_2h_2 + h_{f_3} + m_1h_{f_1} = h_{f_2} + (m_1 + m_2)h_{f_2}$$

$$\mathbf{m_2} = \frac{(h_{f_2} + h_{f_3}) - m_1(h_{f_1} - h_{f_2})}{(h_2 - h_{f_2})}$$

$$= \frac{(467.1 - 289.3) - 0.088\,(640.1 - 467.1)}{(2430 - 467.1)} = \frac{162.57}{1962.9}$$

= **0.0828 kJ/kg of entering steam. (Ans.)**

At heater No. 3 :

$$m_3h_3 + h_{f_5} + (m_1 + m_2)h_{f_2} = h_{f_3} + (m_1 + m_2 + m_3)h_{f_3} \quad ...(i)$$

At drain cooler :

$$(m_1 + m_2 + m_3)h_{f_3} + h_{f_4} = h_{f_5} + (m_1 + m_2 + m_3)h_{f_4}$$

$$\therefore \quad h_{f_5} = (m_1 + m_2 + m_3)\,(h_{f_3} - h_{f_4}) + h_{f_4} \quad ...(ii)$$

Inserting the value of h_{f_5} in eqn. (*i*), we get

$$m_3h_3 + (m_1 + m_2 + m_3)\,(h_{f_3} - h_{f_4}) + h_{f_4} + (m_1 + m_2)\,h_{f_2} = h_{f_3} + (m_1 + m_2 + m_3)h_{f_3}$$

$$\therefore \quad \mathbf{m_3} = \frac{(h_{f_3} - h_{f_4}) - (m_1 + m_2)(h_{f_2} - h_{f_4})}{h_3 - h_{f_4}}$$

$$= \frac{(289.3 - 137.8) - (0.088 + 0.0828)(467.1 - 137.8)}{(2210 - 137.8)}$$

$$= \frac{151.5 - 56.24}{2072.2}$$ = **0.046 kJ/kg of entering steam. (Ans.)**

Work done/kg (*neglecting pump work*)

$= (h_0 - h_1) + (1 - m_1)\,(h_1 - h_2) + (1 - m_1 - m_2)\,(h_2 - h_3) + (1 - m_1 - m_2 - m_3)\,(h_3 - h_4)$

$= (2905 - 2600) + (1 - 0.088)\,(2600 - 2430) + (1 - 0.088 - 0.0828)\,(2430 - 2210)$
$+ (1 - 0.088 - 0.0828 - 0.046)\,(2210 - 2000)$

$= 305 + 155.04 + 182.42 + 164.47 = 806.93$ kJ/kg

Heat supplied/kg $= h_0 - h_{f_1} = 2905 - 640.1 = 2264.9$ kJ/kg.

(*ii*) **Thermal efficiency of the cycle, $\eta_{Thermal}$:**

$$\eta_{Thermal} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{806.93}{2264.9} = \mathbf{0.3563 \text{ or } 35.63\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Thermal efficiency of Rankine cycle, $\eta_{Rankine}$:**

$$\eta_{Rankine} = \frac{h_0 - h_4}{h_0 - h_{f_4}} = \frac{2905 - 2000}{2905 - 137.8} = \mathbf{0.327 \text{ or } 32.7\%.} \quad \textbf{(Ans.)}$$

(*iv*) **Theoretical gain due to regenerative feed heating**

$$= \frac{35.63 - 32.7}{35.63} = \mathbf{0.0822 \text{ or } 8.22\%.} \quad \textbf{(Ans.)}$$

(*v*) **Steam consumption with regenerative feed heating**

$$= \frac{1 \times 3600}{\text{Work done / kg}} = \frac{1 \times 3600}{806.93} = \mathbf{4.46 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

Steam consumption without regenerative feed heating

$$= \frac{1 \times 3600}{\text{Work done / kg without regeneration}} = \frac{1 \times 3600}{h_0 - h_4}$$

$$= \frac{1 \times 3600}{2905 - 2000} = \mathbf{3.97 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

(*vi*) **Quantity of steam passing through the last stage of a 50000 kW turbine with regenerative feed-heating**

$$= 4.46\,(1 - m_1 - m_2 - m_3) \times 50000$$

$$= 4.46\,(1 - 0.088 - 0.0828 - 0.046) \times 50000 = \mathbf{174653.6 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

Same without regenerative arrangement

$$= 3.97 \times 50000 = \mathbf{198500 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

Example 12.17. *A steam turbine plant developing 120 MW of electrical output is equipped with reheating and regenerative feed heating arrangement consisting of two feed heaters—one surface type on H.P. side and other direct contact type on L.P. side. The steam conditions before the steam stop valve are 100 bar and 530°C. A pressure drop of 5 bar takes place due to throttling in valves.*

Steam exhausts from the H.P. turbine at 25 bar. A small quantity of steam is bled off at 25 bar for H.P. surface heater for feed heating and the remaining is reheated in a reheater to 550°C and the steam enters at 22 bar in L.P. turbine for further expansion. Another small quantity of steam is bled off at pressure 6 bar for the L.P. heater and the rest of steam expands up to the back pressure of 0.05 bar. The drain from the H.P. heater is led to the L.P. heater and the combined feed from the L.P. heater is pumped to the high-pressure feed heater and finally to the boiler with the help of boiler feed pump.

The component efficiencies are : Turbine efficiency 85%, pump efficiency 90%, generator efficiency 96%, boiler efficiency 90% and mechanical efficiency 95%. It may be assumed that the feed-water is heated up to the saturation temperature at the prevailing pressure in feed heater. Work out the following :

(*i*) *Sketch the feed heating system and show the process on T-s and h-s diagrams.*

(*ii*) *Amounts of steam bled off.*

(*iii*) *Overall thermal efficiency of turbo-alternator considering pump work.*

(*iv*) *Specific steam consumption in kg/kWh.* **(AMIE Summer, 1998)**

Solution. (*i*) The *schematic arrangement* including feed heating system, and *T-s* and *h-s* diagrams of the process are shown in Figs. 12.21 and 12.22 respectively.

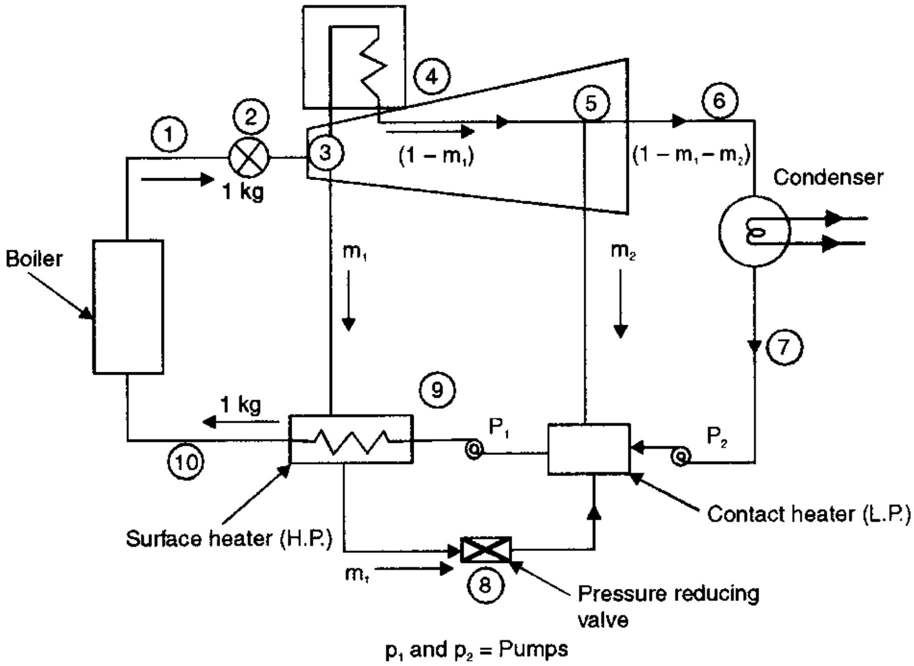


Fig. 12.21

(*ii*) **Amounts of bled off.** The enthalpies at various state points as read from *h-s* diagram/steam tables, in kJ/kg, are :

$$h_1 = h_2 = 3460$$

$h_3' = 3050$, and $\therefore\ h_3 = 3460 - 0.85(3460 - 3050) = 3111.5$

$$h_4 = 3585$$

$h_5' = 3140$, and $\therefore\ h_5 = 3585 - 0.85(3585 - 3140) = 3207$

$h_6' = 2335$, and $\therefore\ h_6 = 3207 - 0.85\ (3207 - 2335) = 2466$

$h_7 = 137.8$ kJ/kg (h_f at 0.05 bar)

$h_8 = h_{10} = 962$ kJ/kg (h_f at 25 bar)

and $h_9 = 670.4$ (h_f at 6 bar).

Enthalpy balance for ***surface heater :***

$m_1h_3 + h_9 = m_1h_8 + h_{10}$, neglecting pump work

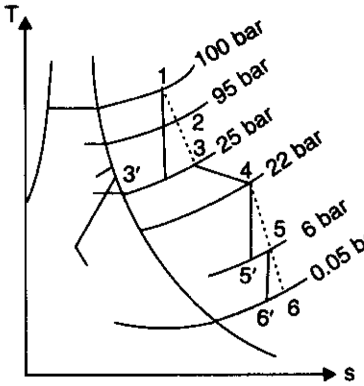
or
$$m_1 = \frac{h_{10} - h_9}{h_3 - h_8} = \frac{962 - 670.4}{3111.5 - 962} = 0.13566 \text{ kg}$$

Enthalpy balance for ***contact heater :***

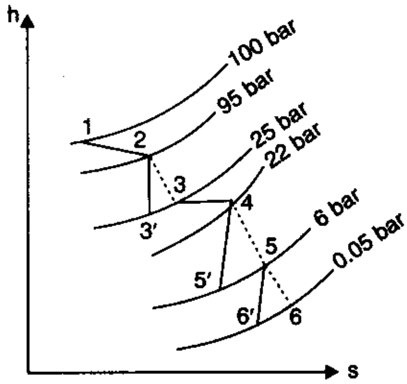
$m_2h_5 + (1 - m_1 - m_2)h_7 + m_1h_8 = h_9$, neglecting pump work

or $m_2 \times 3207 + (1 - 0.13566 - m_2) \times 137.8 + 0.13566 \times 962 = 670.4$

or $m_2 = 0.1371$ kg.



(a) T-s diagram



(b) h-s diagram

Fig. 12.22

Pump Work. Take specific volume of water as 0.001 m³/kg.

$(W_{pump})_{L.P.} = (1 - m_1 - m_2)(6 - 0.05) \times 0.001 \times 10^2$

$= (1 - 0.13566 - 0.1371) \times 5.95 \times 0.1 = 0.4327$ **kJ/kg.**

$(W_{pump})_{H.P.} = 1 \times (100 - 6) \times 0.001 \times 10^2 = 9.4$ **kJ/kg**

Total pump work (actual) $= \dfrac{0.4327 + 9.4}{0.9} = 10.925$ kJ/kg

Turbine output (indicated) $= (h_2 - h_3) + (1 - m_1)(h_4 - h_5) + (1 - m_1 - m_2)(h_5 - h_6)$

$= (3460 - 3111.5) + (1 - 0.13566)(3585 - 3207)$

$+ (1 - 0.13566 - 0.1371)(3207 - 2466)$

$= 1214.105$ kJ/kg

Net electrical output $=$ (Indicated work – Pump work) $\times \eta_{mech.} \times \eta_{gen.}$

$= (1214.105 - 10.925) \times 0.9 \times 0.96 = 1039.55$ kJ/kg

[**Note.** All the above calculations are for 1 kg of main (boiler) flow.]

$\therefore$ Main steam flow rate $= \dfrac{120 \times 10^3 \times 3600}{1039.55} = 4.155 \times 10^5$ kJ/h.

Amounts of bled off are :

(*a*) Surface (high pressure) heater,

$= 0.13566$ kg/kg of boiler flow

or $= 0.13566 \times 4.155 \times 10^5$

i.e., $=$ **5.6367 × 10⁴ kg/h. (Ans.)**

(*b*) Direct contact (low pressure) heater

$= 0.1371$ kg/kg of boiler flow

or $= 0.1371 \times 4.155 \times 10^5$

i.e., $=$ **5.697 × 10⁴ kg/h. (Ans.)**

(*iii*) **Overall thermal efficiency,** $\eta_{overall}$ **:**

Heat input in boiler $= \dfrac{h_1 - h_{10}}{\eta_{boiler}} = \dfrac{3460 - 962}{0.9}$

$= 2775.6$ kJ/kg of boiler flow.

Heat input in reheater $= \dfrac{h_4 - h_3}{\eta_{boiler}} = \dfrac{3585 - 3111.5}{0.9} = 526.1$ kJ/kg of boiler flow

$$\therefore \qquad \eta_{overall} = \frac{1039.55}{2775.6 + 526.1} \times 100 = \mathbf{31.48\%}. \quad \textbf{(Ans.)}$$

(*iv*) **Specific steam consumption :**

$$\text{Specific steam consumption} = \frac{4.155 \times 10^5}{120 \times 10^3} = \mathbf{3.4625\ kg/kWh.} \quad \textbf{(Ans.)}$$

12.5. REHEAT CYCLE

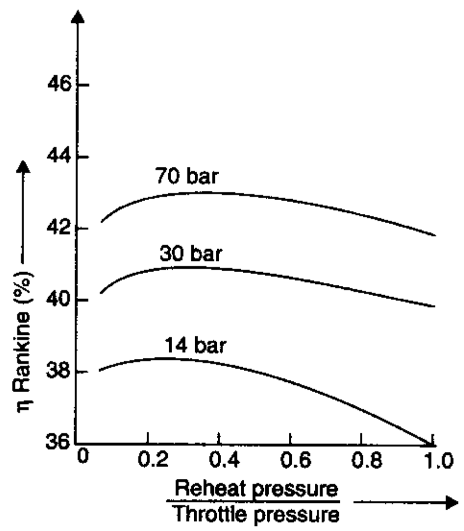
For attaining greater thermal efficiencies when the initial pressure of steam was raised beyond 42 bar it was found that resulting condition of steam after, expansion was increasingly wetter and exceeded in the safe limit of 12 per cent condensation. It, therefore, became necessary to *reheat* the steam after part of expansion was over so that the resulting condition after complete expansion fell within the region of permissible wetness.

The reheating or resuperheating of steam is now universally used when high pressure and temperature steam conditions such as 100 to 250 bar and 500°C to 600°C are employed for throttle. For plants of *still higher pressures and temperatures, a double reheating may be used.*

In actual practice reheat *improves* the cycle efficiency by about 5% for a 85/15 bar cycle. A *second reheat* will give a *much less gain* while the initial cost involved would be so high as to prohibit use of two stage reheat except in case of very high initial throttle conditions. The cost of reheat equipment consisting of boiler, piping and controls may be 5% to 10% more than that of the conventional boilers and this additional expenditure is justified only if gain in thermal efficiency is sufficient to promise a return of this investment. *Usually a plant with a base load capacity of 50000 kW and initial steam pressure of 42 bar would economically justify the extra cost of reheating.*

The improvement in thermal efficiency due to reheat is greatly dependent upon the *reheat pressure* with respect to the original pressure of steam.

Fig. 12.23 shows the reheat pressure selection on cycle efficiency.



Condenser pressure : 12.7 mm Hg
Temperature of throttle and heat : 427°C
Fig. 12.23. Effect of reheat pressure selection on cycle efficiency.

Fig. 12.24 shows a schematic diagram of a theoretical single-stage reheat cycle. The corresponding representation of ideal reheating process on *T-s* and *h-s* chart is shown in Figs. 12.25 (*a* and *b*).

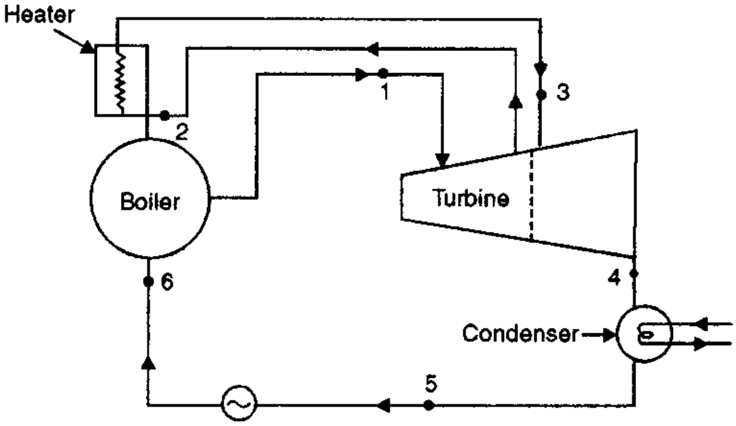


Fig. 12.24. Reheat cycle.

Refer to Fig. 12.25. 5-1 shows the formation of steam in the boiler. The steam as at state point 1 (*i.e.*, pressure p_1 and temperature T_1) enters the turbine and expands isentropically to a certain pressure p_2 and temperature T_2. From this state point 2 the whole of steam is drawn out of the turbine and is reheated in a reheater to a temperature T_3. (Although there is an *optimum pressure* at which the steam should be removed for reheating, if the highest return is to be obtained, yet, for simplicity, the whole steam is removed from the high pressure exhaust, where the pressure is about *one-fifth* of boiler pressure, and after undergoing a 10% pressure drop, in circulating through the heater, it is returned to intermediate pressure or low pressure turbine). This reheated steam is then readmitted to the turbine where it is expanded to condenser pressure isentropically.

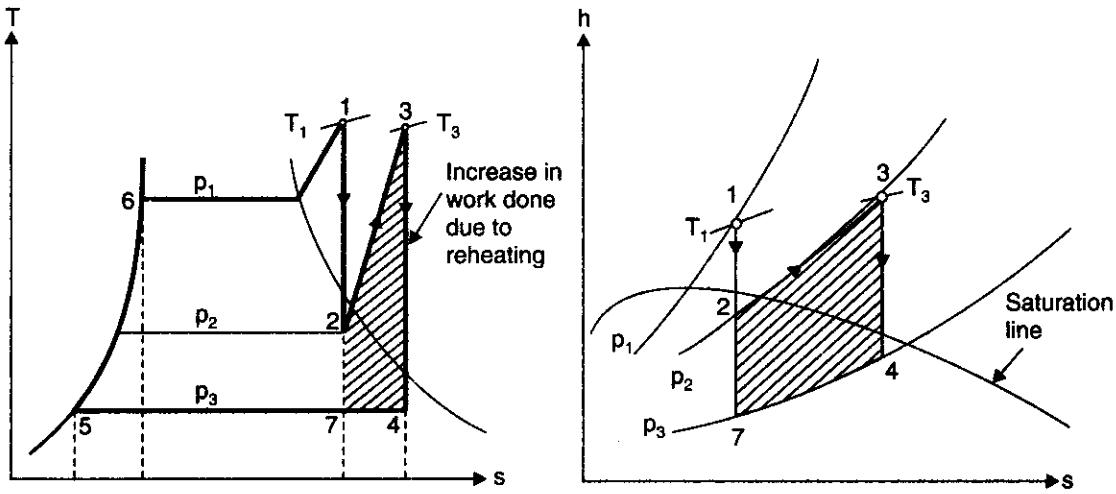


Fig. 12.25. Ideal reheating process on *T-s* and *h-s* chart.

Note. Superheating of steam. The primary object of superheating steam and supplying it to the primemovers is *to avoid too much wetness at the end of expansion*. Use of inadequate degree of superheat in *steam engines would cause greater condensation* in the engine cylinder ; while in case of *turbines* the moisture content of steam would result in *undue blade erosion*. The *maximum wetness in the final condition of steam that may be tolerated without any appreciable harm to the turbine blades is about 12 per cent*. Broadly each *1 per cent* of moisture in steam reduces the efficiency of that part of the turbine in which wet steam passes by 1 per cent to 1.5 per cent and in engines about 2 per cent.

Advantages of superheated steam :

(*i*) Superheating reduces the initial condensation losses in steam engines.

(*ii*) Use of superheated steam results in *improving the plant efficiency* by effecting a *saving in cost of fuel*. This saving may be of the order of 6% to 7% due to first 38°C of superheat and 4% to 5% for next 38°C and so on. This saving results due to the fact that the heat content and consequently the capacity to do work in superheated steam is increased and the quantity of steam required for a given output of power is reduced. Although additional heat has to be added in the boiler there is reduction in the work to be done by the feed pump, the condenser pump and other accessories due to reduction in quantity of steam used. It is estimated that the *quantity of steam may be reduced by 10% to 15% for first 38°C of superheat and somewhat less for the next 38°C of superheat in the case of condensing turbines.*

(*iii*) When a superheater is used in a boiler it helps in *reducing the stack temperatures* by extracting heat from the flue gases before these are passed out of chimney.

Thermal efficiency with '*Reheating*' (neglecting pump work) :

Heat supplied $= (h_1 - h_{f_4}) + (h_3 - h_2)$

Heat rejected $= h_4 - h_{f_4}$

Work done by the turbine = Heat supplied – heat rejected

$$= (h_1 - h_{f_4}) + (h_3 - h_2) - (h_4 - h_{f_4})$$

$$= (h_1 - h_2) + (h_3 - h_4)$$

Thus, theoretical thermal efficiency of reheat cycle is

$$\eta_{\text{thermal}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)} \quad ...(12.11)$$

If pump work, $W_p = \frac{v_f (p_1 - p_b)}{1000}$ kJ/kg is considered, the thermal efficiency is given by :

$$\eta_{\text{thermal}} = \frac{[(h_1 - h_4) + (h_3 - h_4)] - W_p}{[(h_1 - h_{f_4}) + (h_3 - h_2)] - W_p} \quad ...(12.12)$$

W_p is usually small and *neglected*.

Thermal efficiency *without reheating* is

$$\eta_{\text{thermal}} = \frac{h_1 - h_7}{h_1 - h_{f_4}} \ (\because \ h_{f_4} = h_{f_7}) \quad ...(12.13)$$

Note 1. The reheater may be incorporated in the walls of the main boiler ; it may be a separately fired superheater or it may be heated by a coil carrying high-pressure superheated steam, this system being analogous to a steam jacket.

2. Reheating should be done at '*optimum pressure*' because if the steam is reheated early in its expansion then the additional *quantity of heat supplied will be small* and thus thermal efficiency gain will be small ; and if the reheating is done at a fairly low pressure, then, although a large amount of additional heat is supplied, the steam will have a high degree of superheat (as is clear from Mollier diagram), thus a large proportion of the heat supplied in the reheating process *will be thrown to waste in the condenser*.

Advantages of 'Reheating' :

1. There is an increased output of the turbine.
2. Erosion and corrosion problems in the steam turbine are eliminated/avoided.
3. There is an improvement in the thermal efficiency of the turbines.
4. Final dryness fraction of steam is improved.
5. There is an increase in the nozzle and blade efficiencies.

Disadvantages :

1. Reheating requires more maintenance.
2. The increase in thermal efficiency is not appreciable in comparison to the expenditure incurred in reheating.

☞**Example 12.18.** *Steam at a pressure of 15 bar and 250°C is expanded through a turbine at first to a pressure of 4 bar. It is then reheated at constant pressure to the initial temperature of 250°C and is finally expanded to 0.1 bar. Using Mollier chart, estimate the work done per kg of steam flowing through the turbine and amount of heat supplied during the process of reheat. Compare the work output when the expansion is direct from 15 bar to 0.1 bar without any reheat. Assume all expansion processes to be isentropic.*

Solution. Refer Fig. 12.26.

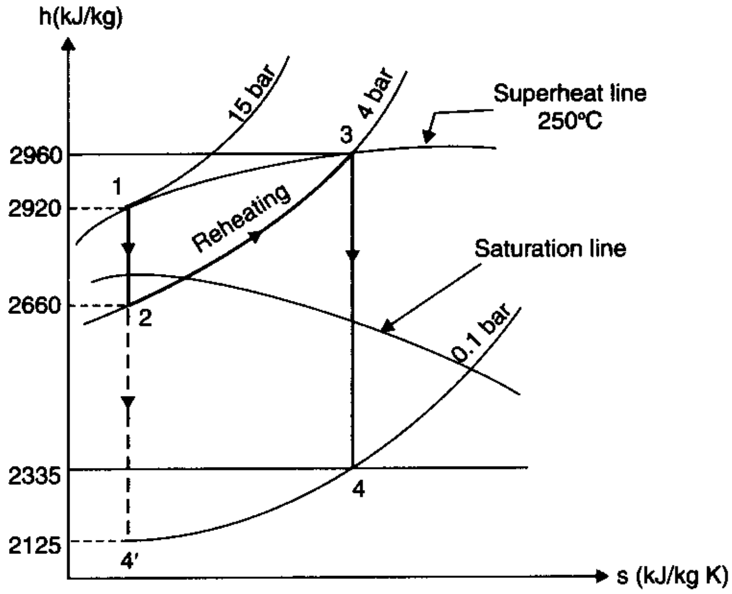


Fig. 12.26

Pressure, $p_1 = 15$ bar ;

$p_2 = 4$ bar ;

$p_4 = 0.1$ bar.

Work done per kg of steam,

W = Total heat drop

$= [(h_1 - h_2) + (h_3 - h_4)]$ kJ/kg ...(*i*)

Amount of heat supplied during process of reheat,

$$h_{\text{reheat}} = (h_3 - h_2) \text{ kJ/kg} \quad ...(ii)$$

From Mollier diagram or h-s chart,

$$h_1 = 2920 \text{ kJ/kg}, \; h_2 = 2660 \text{ kJ/kg}$$
$$h_3 = 2960 \text{ kJ/kg}, \; h_2 = 2335 \text{ kJ/kg}$$

Now, by putting the values in eqns. (*i*) and (*ii*), we get

$$W = (2920 - 2660) + (2960 - 2335)$$
$$= \mathbf{885 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

Hence *work done per kg of steam* = **885 kJ/kg. (Ans.)**

Amount of heat supplied during reheat,

$$h_{\text{reheat}} = (2960 - 2660) = \mathbf{300 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

If the expansion would have been continuous without reheating *i.e.,* 1 to 4′, the work output is given by

$$W_1 = h_1 - h_4'$$

From Mollier diagram,

$$h_{4'} = 2125 \text{ kJ/kg}$$

$$\therefore \quad W_1 = 2920 - 2125 = \mathbf{795 \text{ kJ/kg.}} \quad \textbf{(Ans.)}$$

Example 12.19. *A steam power plant operates on a theoretical reheat cycle. Steam at boiler at 150 bar, 550°C expands through the high pressure turbine. It is reheated at a constant pressure of 40 bar to 550°C and expands through the low pressure turbine to a condenser at 0.1 bar. Draw T-s and h-s diagrams. Find :*

(*i*) *Quality of steam at turbine exhaust* ; (*ii*) *Cycle efficiency* ;

(*iii*) *Steam rate in kg/kWh.* **(AMIE Summer, 1999)**

Solution. Refer Figs. 12.27 and 12.28.

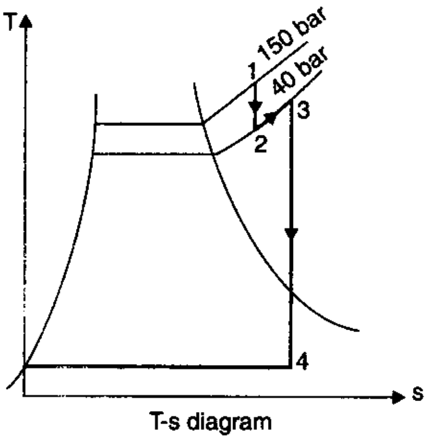


Fig. 12.27

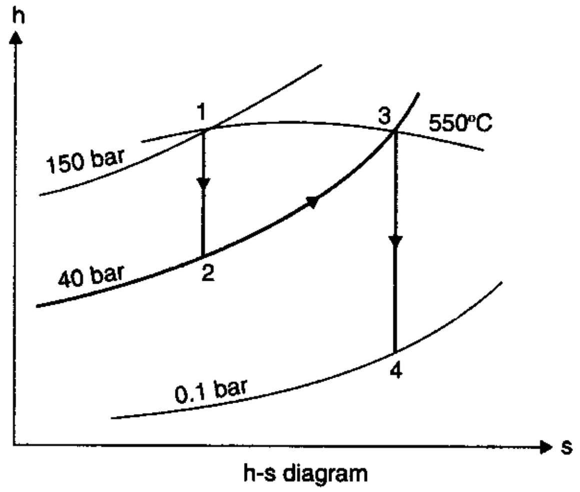


Fig. 12.28

From Mollier diagram (h-s diagram) :

$$h_1 = 3450 \text{ kJ/kg} \; ; \; h_2 = 3050 \text{ kJ/kg} \; ; \; h_3 = 3560 \text{ kJ/kg} \; ; \; h_4 = 2300 \text{ kJ/kg}$$

$$h_{f_4} \text{ (from steam tables, at 0.1 bar)} = 191.8 \text{ kJ/kg}$$

(*i*) **Quality of steam at turbine exhaust, x_4 :**

x_4 = 0.88 (From Mollier diagram)

(*ii*) **Cycle efficiency, η_{cycle} :**

$$\eta_{cycle} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)}$$

$$= \frac{(3450 - 3050) + (3560 - 2300)}{(3450 - 191.8) + (3560 - 3050)} = \frac{1660}{3768.2} = \mathbf{0.4405 \text{ or } 44.05\%.} \quad \textbf{(Ans.)}$$

(*iii*) **Steam rate in kg/kWh :**

$$\text{Steam rate} = \frac{3600}{(h_1 - h_2) + (h_3 - h_4)} = \frac{3600}{(3450 - 3050) + (3560 - 2300)}$$

$$= \frac{3600}{1660} = \mathbf{2.17 \text{ kg/kWh.}} \quad \textbf{(Ans.)}$$

Example 12.20. *A turbine is supplied with steam at a pressure of 32 bar and a temperature of 410°C. The steam then expands isentropically to a pressure of 0.08 bar. Find the dryness fraction at the end of expansion and thermal efficiency of the cycle.*

If the steam is reheated at 5.5 bar to a temperature of 395°C and then expanded isentropically to a pressure of 0.08 bar, what will be the dryness fraction and thermal efficiency of the cycle ?

Solution. First case. Refer Fig. 12.29.

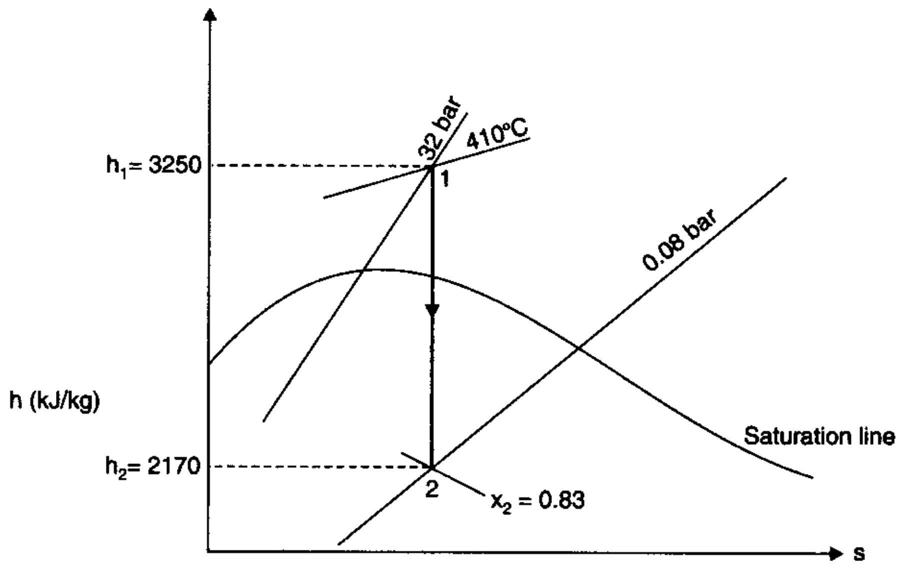


Fig. 12.29

From Mollier chart :

h_1 = 3250 kJ/kg

h_2 = 2170 kJ/kg

Heat drop (or work done) = $h_1 - h_2$

= 3250 − 2170 = 1080 kJ/kg

Heat supplied $= h_1 - h_{f_2}$

$= 3250 - 173.9$ $[h_{f_2} = 173.9$ kJ/kg at 0.08 bar]

$= 3076.1$ kJ/kg

Thermal efficiency $= \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{1080}{3076.1}$ = **0.351 or 35.1%. (Ans.)**

Exhaust steam condition, x_2

= **0.83 (From Mollier chart). (Ans.)**

Second case. Refer Fig. 12.30 (*b*).

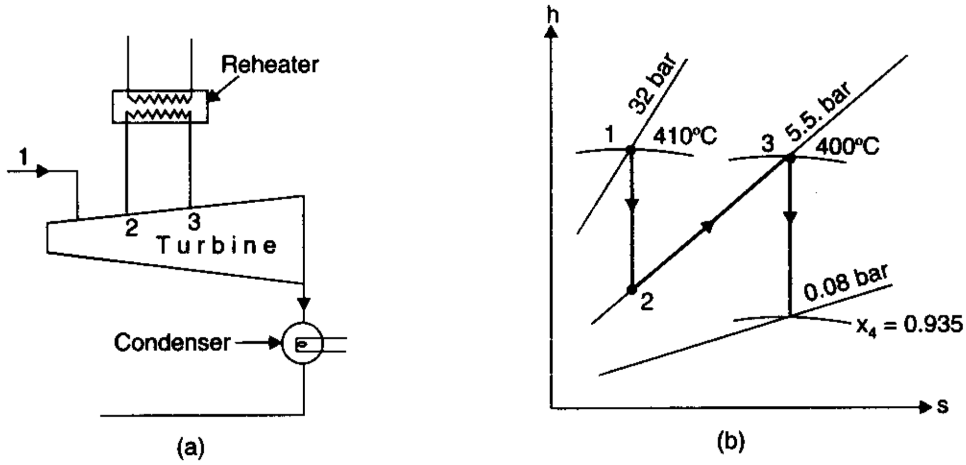


Fig. 12.30

From Mollier chart :

$h_1 = 3250$ kJ/kg ;
$h_2 = 2807$ kJ/kg ;
$h_3 = 3263$ kJ/kg ;
$h_4 = 2426$ kJ/kg.

Work done $= (h_1 - h_2) + (h_3 - h_4) = (3250 - 2807) + (3263 - 2426) = 1280$ kJ/kg

Heat supplied $= (h_1 - h_{f_4}) + (h_3 - h_2)$

$= (3250 - 173.9) + (3263 - 2807) = 3532$ kJ/kg

Thermal efficiency $= \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{1280}{3532}$ = **0.362 or 36.2%. (Ans.)**

Condition of steam at the exhaust,

x_4 = 0.935 [*From Mollier chart*]. **(Ans.)**

Example 12.21. (*a*) *How does erosion of turbine blades occur ? State the methods of preventing erosion of turbine blades.*

(*b*) *What do you mean by TTD of a feed water heater ? Draw temperature-path-line diagram of a closed feed water heater used in regenerative feed heating cycle.*

(c) *In a 15 MW steam power plant operating on ideal reheat cycle, steam enters the H.P. turbine at 150 bar and 600°C. The condenser is maintained at a pressure of 0.1 bar. If the moisture content at the exit of the L.P. turbine is 10.4%, determine :*

(i) *Reheat pressure ;* (ii) *Thermal efficiency ;* (iii) *Specific steam consumption ; and* (iv) *Rate of pump work in kW. Assume steam to be reheated to the initial temperature.*

(AMIE Summer, 1998)

Solution. (a) The erosion of the moving blades is *caused by the presence of water particles in (wet) steam in the L.P. stages.* The water particles strike the leading surface of the blades. Such impact, if sufficiently heavy, produces *severe local stresses in the blade material causing the surface metal to fail and flake off.*

The erosion, if any, is more likely to occur in the region where the steam is wettest, *i.e.*, in the last one or two stages of the turbine. Moreover, the water droplets are concentrated in the outer parts of the flow annuals where the velocity of impact is highest.

Erosion difficulties due to moisture in the steam may be avoided by reheating (see Fig. 12.31). The whole of steam is taken from the turbine at a suitable point 2, and a further supply of heat is given to it along 2-3 after which the steam is readmitted to the turbine and expanded along 3-4 to condenser pressure.

Erosion *may also be reduced by using steam traps in between the stages to separate moisture from the steam.*

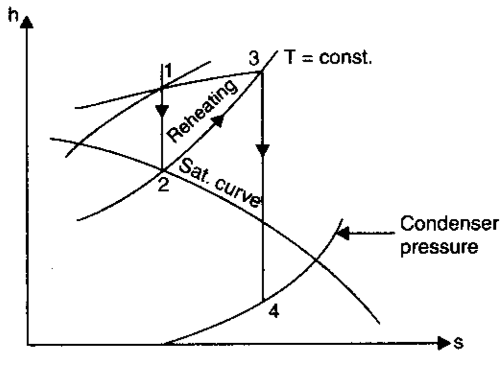


Fig. 12.31

(b) TTD means *"Terminal temperature difference"*. It is the *difference between temperatures of bled steam/condensate and the feed water at the two ends of the feed water heater.*

The required temperature-path-line diagram of a closed feed water heater is shown in Fig. 12.32.

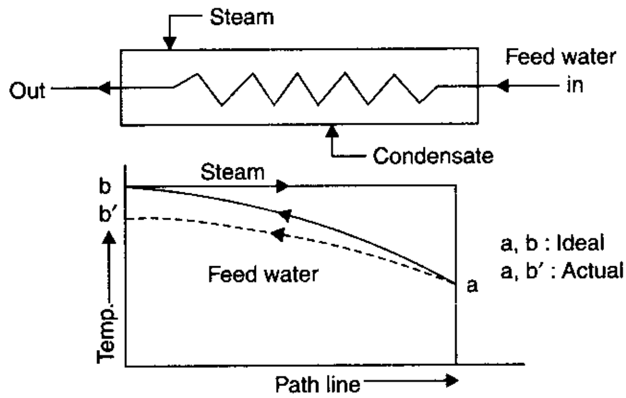


Fig. 12.32

(c) The cycle is shown on *T-s* and *h-s* diagrams in Figs. 12.33 and 12.34 respectively. The following values are read from the Mollier diagram :

$$h_1 = 3580 \text{ kJ/kg}, \; h_2 = 3140 \text{ kJ/kg}, \; h_3 = 3675 \text{ kJ/kg}, \text{ and } h_4 = 2335 \text{ kJ/kg}$$

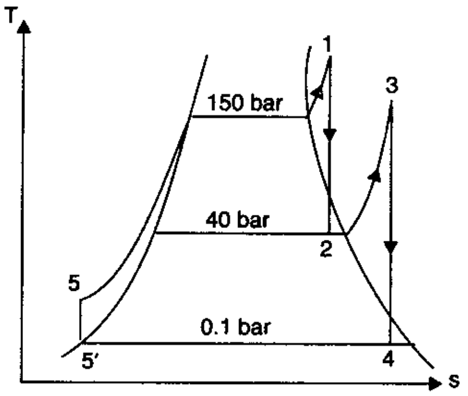


Fig. 12.33

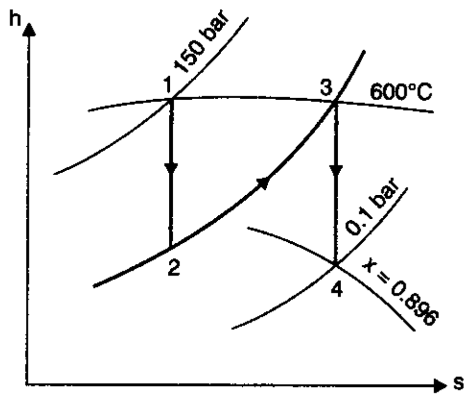


Fig. 12.34

Moisture contents in exit from L.P. turbine = 10.4%

$$x_4 = 1 - 0.104 = 0.896$$

(*i*) **Reheat pressure :** From the Mollier diagram, the **reheat pressure is 40 bar.** **(Ans.)**

(*ii*) **Thermal efficiency, η_{th} :**

Turbine work $= (h_1 - h_2) + (h_3 - h_4)$

$= (3580 - 3140) + (3675 - 2335) = 1780$ kJ/kg.

Assuming specific volume of water $= 10^{-3}$ m^3/kg, the pump work $= 10^{-3}$ $(150 - 0.1)$ $= 0.15$ kJ/kg, *i.e.*, may be neglected in computing of η_{th}, $h_5 = h_4 = 191.8$ kJ/kg, (h_f at 0.1 bar) from steam tables,

$$Q_{input} = (h_1 - h_5) + (h_3 - h_2)$$

$$= (3580 - 191.8) + (3675 - 3140) = 3923.2 \text{ kJ/kg}$$

$$\%\eta_{th} = \frac{1780}{3923.2} \times 100 = \mathbf{45.37\%}. \quad \textbf{(Ans.)}$$

(*iii*) **Specific steam consumption :**

Steam consumption $= \dfrac{15 \times 10^3}{1780} = 8.427$ kg/s

Specific steam consumption $= \dfrac{8.427 \times 3600}{15 \times 10^3} =$ **2.0225 kg/kWh. (Ans.)**

(*iv*) **Rate of pump work :**

Rate of pump work $= 8.427 \times 0.15 =$ **1.26 kW. (Ans.)**

12.6. BINARY VAPOUR CYCLE

Carnot cycle gives the highest thermal efficiency which is given by $\dfrac{T_1 - T_2}{T_1}$. To approach this cycle in an actual engine it is necessary that whole of heat must be supplied at constant temperature T_1 and rejected at T_2. This can be achieved only by using a vapour in the wet field but not in the superheated. The efficiency depends on temperature T_1 since T_2 is fixed by the natural sink to which heat is rejected. This means that T_1 should be *as large as possible, consistent with the vapour being saturated.*

If we use steam as the working medium the temperature rise is accompanied by rise in pressure and at critical temperature of 374.15°C the pressure is as high as 225 bar which will create many difficulties in design, operation and control. It would be desirable to use some fluid other than steam which has more desirable thermodynamic properties than water. An ideal fluid for this purpose should have a *very high critical temperature combined with low pressure*. Mercury, diphenyl oxide and similar compounds, aluminium bromide and zinc ammonium chloride are fluids which possess the required properties in varying degrees. Mercury is the only working fluid which has been successfully used in practice. It has high critical temperature (588.4°C) and correspondingly low critical pressure (21 bar abs.). *The mercury alone cannot be used as its saturation temperature at atmospheric pressure is high (357°C). Hence* ***binary vapour cycle*** *is generally used to increase the overall efficiency of the plant.* Two fluids (mercury and water) are used in cascade in the binary cycle for production of power.

The *few more properties* required for an ideal binary fluid used in high temperature limit are listed below :

1. It should have high critical temperature at reasonably low pressure.
2. It should have high heat of vaporisation to keep the weight of fluid in the cycle to minimum.
3. Freezing temperature should be below room temperature.
4. It should have chemical stability through the working cycle.
5. It must be non-corrosive to the metals normally used in power plants.
6. It must have an ability to wet the metal surfaces to promote the heat transfer.
7. The vapour pressure at a desirable condensation temperature should be nearly atmospheric which will eliminate requirement of power for maintenance of vacuum in the condenser.
8. After expansion through the primemover the vapour should be nearly saturated so that a desirable heat transfer co-efficient can be obtained which will reduce the size of the condenser required.
9. It must be available in large quantities at reasonable cost.
10. It should not be toxic and, therefore, dangerous to human life.

Although mercury does not have all the required properties, it is more favourable than any other fluid investigated. It is most stable under all operating conditions.

Although, mercury does not cause any corrosion to metals, but it is extremely dangerous to human life, therefore, elaborate precautions must be taken to prevent the escape of vapour. The *major disadvantage* associated with mercury is that it *does not wet surface of the metal and forms a serious resistance to heat flow*. This difficulty can be considerably reduced by adding magnesium and titanium (2 parts in 100000 parts) in mercury.

Thermal properties of mercury :

Mercury fufills practically all the *desirable thermodynamic properties* stated above.

1. Its freezing point is – 3.3°C and boiling point is – 354.4°C at atmospheric pressure.
2. The pressure required when the temperature of vapour is 540°C is only 12.5 bar (app.) and, therefore, heavy construction is not required to get high initial temperature.
3. Its liquid saturation curve is very steep, approaching the isentropic of the Carnot cycle.
4. It has no corrosive or erosive effects upon metals commonly used in practice.
5. Its critical temperature is so far removed from any possible upper temperature limit with existing metals as to cause no trouble.

Some *undesirable properties* of mercury are listed below :

1. Since the latent heat of mercury is quite low over a wide range of desirable condensation temperatures, therefore, several kg of mercury must be circulated per kg of water evaporated in binary cycle.
2. The cost is a considerable item as the quantity required is 8 to 10 times the quantity of water circulated in binary system.
3. Mercury vapour in larger quantities is poisonous, therefore, the system must be perfect and tight.

Fig. 12.35 shows the schematic line diagram of binary vapour cycle using mercury and water as working fluids. The processes are represented on T-s diagram as shown in Fig. 12.36.

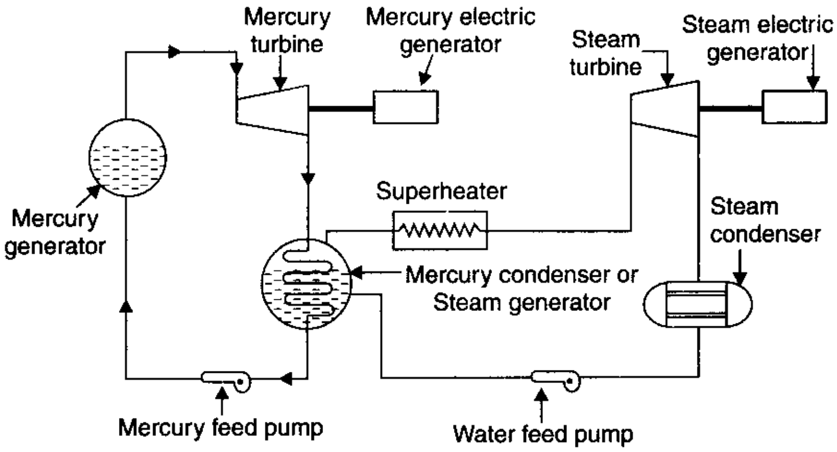


Fig. 12.35. Line diagram of binary vapour cycle.

Analysis of Binary vapour cycle :

h_{hg_1} = Heat supplied per kg of Hg (mercury) vapour formed in the mercury boiler.

h_{hg_2} = Heat lost by one kg of Hg vapour in the mercury condenser.

h_s = Heat given per kg of steam generated in the mercury condenser or steam boiler.

W_{hg} = Work done per kg of Hg in the cycle.

W_s = Work done per kg of steam in the steam cycle.

η_s = Thermal efficiency of the steam cycle.

η_{hg} = Thermal efficiency of the Hg cycle.

m = Mass of Hg in the Hg cycle per kg of steam circulated in the steam cycle.

The heat losses to the surroundings, in the following analysis, are neglected and steam generated is considered one kg and Hg in the circuit is m kg per kg of water in the steam cycle.

Heat supplied in the Hg boiler

$$h_t = m \times h_{hg_1} \quad \text{...(12.14)}$$

Work done in the mercury cycle

$$= m \,.\, W_{hg} \quad \text{...(12.15)}$$

Work done in the steam cycle

$$= 1 \times W_s \quad \text{...(12.16)}$$

Total work done in the binary cycle is given by

$$W_t = m\,W_{hg} + W_s \qquad ...(12.17)$$

∴ Overall efficiency of the binary cycle is given by

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W_t}{h_t} = \frac{mW_{hg} + W_s}{mh_{hg_1}} \qquad ...(12.18)$$

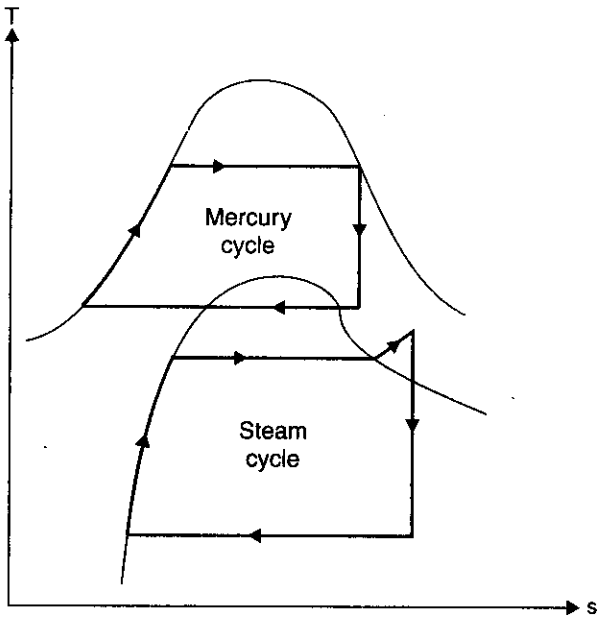


Fig. 12.36. Binary vapour cycle on T-s diagram.

Thermal efficiency of the mercury cycle is given by

$$\eta_{hg} = \frac{mW_{hg}}{mh_{hg_1}}$$

$$= \frac{W_{hg}}{h_{hg_1}} = \frac{h_{hg_1} - h_{hg_2}}{h_{hg_1}} = 1 - \frac{h_{hg_2}}{h_{hg_1}} \qquad ...(12.19)$$

$$= \frac{mh_{hg_1} - h_s}{mh_{hg_1}} = 1 - \frac{1}{m} \cdot \frac{h_s}{h_{hg_1}} \qquad ...(12.20)$$

Heat lost by mercury vapour = Heat gained by steam

$$\therefore \qquad m\,h_{hg_2} = 1 \times h_s \qquad ...(12.21)$$

Substituting the value of m from eqn. (12.21) into eqn. (12.20), we get

$$\eta_{hg} = 1 - \frac{h_{hg_2}}{h_{hg_1}} \qquad ...(12.22)$$

The thermal efficiency of the steam cycle is given by

$$\eta_s = \frac{W_s}{h_s} = \frac{h_{s_1} - h_{s_2}}{h_{s_1}} = \frac{h_{s_1} - h_{s_2}}{mh_{hg_2}} \qquad ...(12.23)$$

From the eqns. (12.18), (12.20), (12.21), (12.22) and (12.23), we get

$$\eta = \eta_{hg}(1 - \eta_s) + \eta_s \quad ...(12.24)$$

To solve the problems eqns. (12.19), (12.23), (12.24) are used.

In the design of binary cycle, another important problem is the limit of exhaust pressure of the mercury (location of optimum exhaust pressure) which will provide maximum work per kg of Hg circulated in the system and high thermal efficiency of the cycle. It is not easy to decide as number of controlling factors are many.

☞**Example 12.22.** *A binary vapour cycle operates on mercury and steam. Standard mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhausts at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar.*

(i) Determine the overall efficiency of the cycle.

(ii) If 48000 kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine ?

(iii) Assuming that all processes are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow ?

(iv) If the steam leaving the mercury condenser is superheated to a temperature of 300°C in a superheater located in the mercury boiler and if the internal efficiencies of the mercury and steam turbines are 0.84 and 0.88 respectively, calculate the overall efficiency of the cycle. The properties of standard mercury are given below :

p (bar)	*t (°C)*	h_f *(kJ/kg)*	h_g *(kJ/kg)*	s_f *(kJ/kg K)*	s_g *(kJ/kg K)*	v_f *(m³/kg)*	v_g *(m³/kg)*
4.5	*450*	*62.93*	*355.98*	*0.1352*	*0.5397*	79.9×10^{-6}	*0.068*
0.04	*216.9*	*29.98*	*329.85*	*0.0808*	*0.6925*	76.5×10^{-6}	*5.178.*

Solution. The binary vapour cycle is shown in Fig. 12.37.

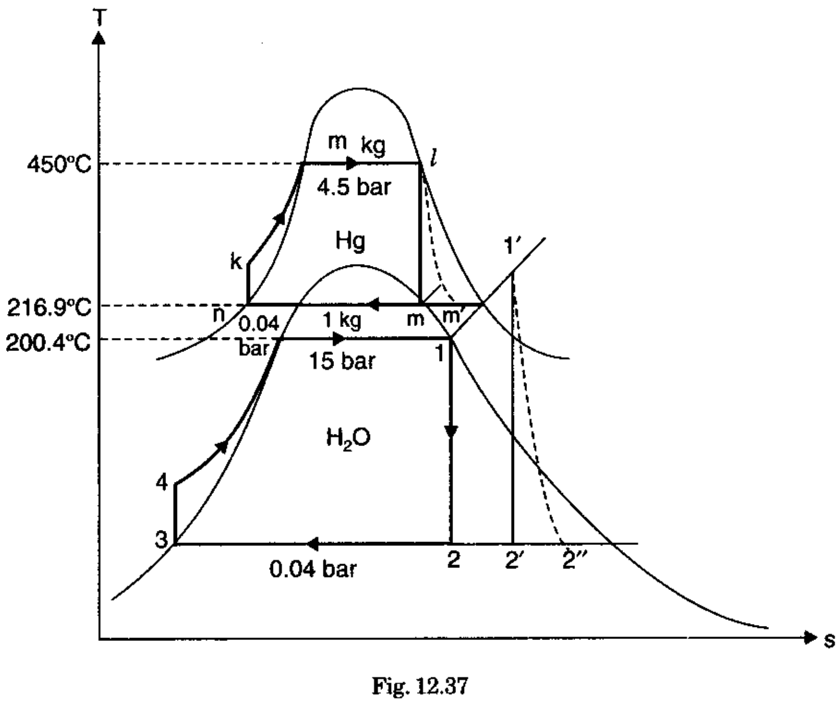


Fig. 12.37

Mercury cycle :

$$h_l = 355.98 \text{ kJ/kg}$$

$$s_l = 0.5397 = s_m = s_f + x_m\, s_{fg}$$

or $$0.5397 = 0.0808 + x_m = (0.6925 - 0.0808)$$

$\therefore$ $$x_m = \frac{(0.5397 - 0.0808)}{(0.6925 - 0.0808)} = 0.75$$

$$h_m = h_f + x_m\, h_{fg} = 29.98 + 0.75 \times (329.85 - 29.98)$$
$$= 254.88 \text{ kJ/kg}$$

Work obtained from mercury turbine

$$(W_T)_{Hg} = h_l - h_m = 355.98 - 254.88 = 101.1 \text{ kJ/kg}$$

Pump work in mercury cycle,

$$(W_P)_{Hg} = h_{f_k} - h_{f_n} = 76.5 \times 10^{-6} \times (4.5 - 0.04) \times 100 = 0.0341 \text{ kJ/kg}$$

$\therefore$ $$W_{net} = 101.1 - 0.0341 \simeq 101.1 \text{ kJ/kg}$$

$$Q_1 = h_l - h_{f_k} = 355.98 - 29.98 = 326 \text{ kJ/kg} \qquad (\because\ h_{f_n} \simeq h_{f_k})$$

$\therefore$ $$\eta_{Hg\ cycle} = \frac{W_{net}}{Q_1} = \frac{101.1}{326} = 0.31 \text{ or } 31\%.$$

Steam cycle :

At 15 bar : $h_1 = 2789.9$ kJ/kg, $s_1 = 6.4406$ kJ/kg

At 0.04 bar : $h_f = 121.5$ kJ/kg, $h_{f_g} = 2432.9$ kJ/kg,

$s_f = 0.432$ kJ/kg K, $s_{fg_2} = 8.052$ kJ/kg K, $v_f = 0.0001$ kJ/kg K

Now, $$s_1 = s_2$$

$$6.4406 = s_f + x_2\, s_{fg} = 0.423 + x_2 \times 8.052$$

$\therefore$ $$x_2 = \frac{6.4406 - 0.423}{8.052} = 0.747$$

$$h_2 = h_{f_2} + x_2\, h_{f_g} = 121.5 + 0.747 \times 2432.9 = 1938.8 \text{ kJ/kg}$$

Work obtained from steam turbine,

$$(W_T)_{steam} = h_1 - h_2 = 2789.9 - 1938.8 = 851.1 \text{ kJ/kg}$$

Pump work in steam cycle,

$$(W_P)_{steam} = h_{f_4} - h_{f_3} = 0.001\,(15 - 0.04) \times 100 = 1.496 \text{ kJ/kg} \simeq 1.5 \text{ kJ/kg}$$

or $$h_{f_4} = h_{f_3} + 1.5 = 121.5 + 1.5 = 123 \text{ kJ/kg}$$

$$Q_1 = h_1 - h_{f_4} = 2789.9 - 123 = 2666.9 \text{ kJ/kg}$$

$$(W_{net})_{steam} = 851.1 - 1.5 = 849.6 \text{ kJ/kg}$$

$\therefore$ $$\eta_{steam\ cycle} = \frac{W_{net}}{Q_1} = \frac{849.6}{2666.6} = 0.318 \text{ or } 31.8\%.$$

(*i*) **Overall efficiency of the binary cycle :**

Overall efficiency of the binary cycle

$$= \eta_{Hg_{cycle}} + \eta_{steam\ cycle} - \eta_{Hg_{cycle}} \times \eta_{steam\ cycle}$$

$$= 0.31 + 0.318 - 0.31 \times 0.318 = 0.5294 \text{ or } 52.94\%$$

Hence *overall efficiency of the binary cycle* = **52.94%. (Ans.)**

$\eta_{overall}$ can also be found out as follows :

Energy balance for a mercury condenser-steam boiler gives :

$$m\,(h_m - h_{f_n}) = 1(h_1 - h_{f_4})$$

where m is the amount of mercury circulating for 1 kg of steam in the bottom cycle

$$\therefore \quad m = \frac{h_1 - h_{f_4}}{h_m - h_{f_n}} = \frac{2666.9}{254.88 - 29.98} = 11.86 \text{ kg}$$

$$(Q_1)_{total} = m\,(h_l - h_{f_k}) = 11.86 \times 326 = 3866.36 \text{ kJ/kg}$$

$$(W_T)_{total} = m\,(h_l - h_m) + (h_1 - h_2)$$

$$= 11.86 \times 101.1 + 851.1 = 2050.1 \text{ kJ/kg}$$

$(W_P)_{total}$ may be *neglected*

$$\eta_{overall} = \frac{W_T}{Q_1} = \frac{2050.1}{3866.36} = 0.53 \text{ or } 53\%.$$

(*ii*) **Flow through mercury turbine :**

If 48000 kg/h of steam flows through the steam turbine, the flow rate of mercury,

$$m_{Hg} = 48000 \times 11.86 = \mathbf{569280 \text{ kg/h.}} \quad \textbf{(Ans.)}$$

(*iii*) **Useful work in binary vapour cycle :**

Useful work, $(W_T)_{total} = 2050.1 \times 48000 = 9840.5 \times 10^4$ kJ/h

$$= \frac{9840.5 \times 10^4}{3600} = 27334.7 \text{ kW} = \mathbf{27.33 \text{ MW.}} \quad \textbf{(Ans.)}$$

(*iv*) **Overall efficiency under new conditions :**

Considering the efficiencies of turbines, we have :

$$(W_T)_{Hg} = h_l - h_m' = 0.84 \times 101.1 = 84.92 \text{ kJ/kg}$$

$$\therefore \quad h_{m'} = h_l - 84.92 = 355.98 - 84.92 = 271.06 \text{ kJ/kg}$$

$$\therefore \quad m'\,(h_{m'} - h_{n'}) = (h_1 - h_{f_4})$$

or

$$m' = \frac{h_1 - h_{f_4}}{h_{m'} - h_{n'}} = \frac{2666.9}{271.06 - 29.98} = 11.06 \text{ kg}$$

$$(Q_1)_{total} = m'\,(h_l - h_{f_k}) + 1\,(h_1' - h_1)$$

[***At 15 bar, 300°C :*** h_g = 3037.6 kJ/kg, s_g = 6.918 kJ/kg K]

$$= 11.06 \times 326 + (3037.6 - 2789.9) = 3853.26 \text{ kJ/kg}$$

$$s_1' = 6.918 = s_2' = 0.423 + x_2' \times 8.052$$

$$\therefore \quad x_2' = \frac{6.918 - 0.423}{8.052} = 0.80.$$

$$h_2' = 121.5 + 0.807 \times 2432.9 = 2084.8 \text{ kJ/kg}$$

$$(W_T)_{steam} = h_1' - h_2' = 0.88\,(3037.6 - 2084.8)$$

$$= 838.46 \text{ kJ/kg}$$

$$(W_T)_{total} = 11.06 \times 84.92 + 838.46 = 1777.67 \text{ kJ/kg}$$

Neglecting pump work,

$$\eta_{overall} = \frac{1777.67}{3853.26} = \mathbf{0.4613 \text{ or } 46.13\%.} \quad \textbf{(Ans.)}$$

ADDITIONAL / TYPICAL EXAMPLES

Example 12.23. *The following data relate to a regenerative steam power plant generating 22500 kW energy, the alternator directly coupled to steam turbine :*

Condition of steam supplied to the steam turbine	*... 60 bar, 450°C*
Condenser vacuum	*... 707.5 mm*
Pressure at which steam is bled from the steam turbine	*... 3 bar*
Turbine efficiency of each portion of expansion	*... 87 per cent*
Boiler efficiency	*... 86 per cent*
Alternator efficiency	*... 94 per cent*
Mechanical efficiency from turbine to generator	*... 97 per cent*

Neglecting the pump work in calculating the input to the boiler, determine :

(i) The steam bled per kg of steam supplied to the turbine.

(ii) The steam generated per hour if the 9 percent of the generator output is used to run the pumps.

(iii) The overall efficiency of the plant.

Solution. The schematic arrangement of the steam power plant is shown in Fig. 12.38 (*a*), while the conditions of the fluid passing through the components are represented on *T-s* and *h-s* diagrams as shown in Figs. 12.38 (*b*) and (*c*). The conditions of the fluid entering and leaving the pump are shown by the same point as the rise in temperature due to pump work is neglected.

Given : Power generated = 22500 kW ;

$$p_1 = 60 \text{ bar} ; t_1 = 450°\text{C} ; p_2 (= p_2') = 3 \text{ bar} ;$$

$$p_3 (= p_3') = \frac{760 - 707.5}{760} \times 1.013 = 0.07 \text{ bar} ; \eta_{turbine\ (each\ portion)} = 87\% ;$$

$$\eta_{boiler} = 86\% ; \eta_{alt.} = 94\%, \eta_{mech.} = 97\%$$

- Locate point 1 corresponding to the conditions : p_1 = 60 bar ; t_1 = 450°C on the *h-s* chart *(Mollier chart).*

  From *h-s* chart ; we find : h_1 = 3300 kJ/kg.

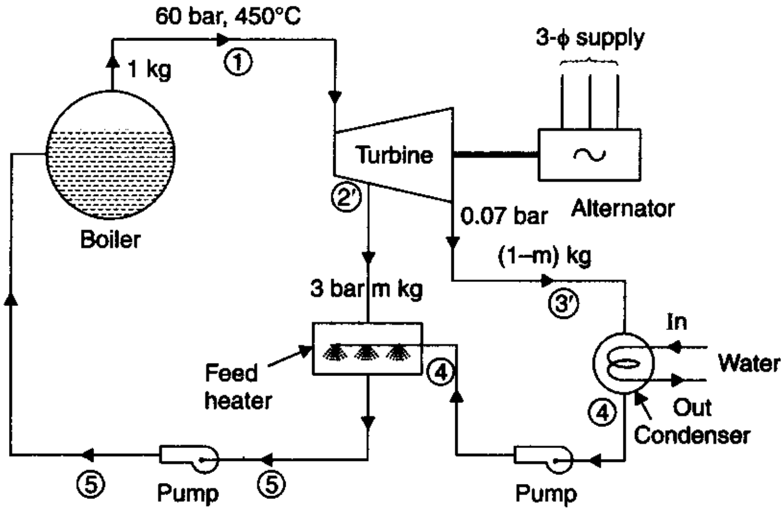
- Draw vertical line through point 1 till it cuts the 3 bar pressure line, then locate point 2.

  $\therefore \quad h_2 = 2607$ kJ/kg

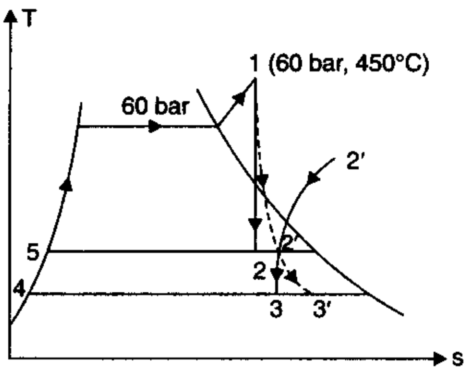
  Now, $\eta_{turbine} = 0.87 = \dfrac{h_1 - h_2'}{h_1 - h_2}$ or $0.87 = \dfrac{3300 - h_2'}{3300 - 2607}$

  $\therefore \quad h_2' = 2697$ kJ/kg

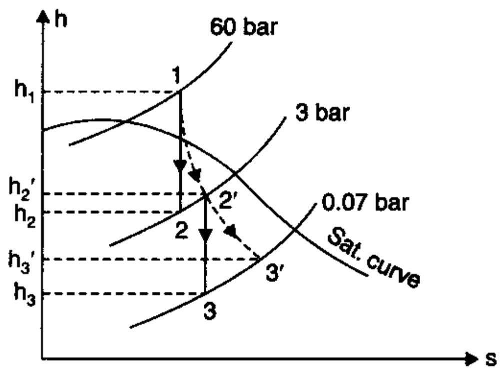
- Locate the point 2 on the *h-s* chart as enthalpy and pressure are known and then draw a vertical line through the point 2 till it cuts the 0.07 bar pressure line and then locate the point 3.



(a) Schematic arrangement of the steam power plant



(b) T-s diagram (Neglecting pump work)



(c) h-s diagram

Fig. 12.38

∴ $h_3 = 2165$ kJ/kg

Again, $\eta_{turbine} = 0.87 = \dfrac{h_2' - h_3'}{h_2' - h_3}$ or $0.87 = \dfrac{2697 - h_3'}{2697 - 2165}$

∴ $h_3' = 2234$ kJ/kg

From steam tables, corresponding to pressures 3 bar and 0.02 bar, the saturated liquid heats at points 4 and 5 are :

$$h_{f4} = 163.4 \text{ kJ/kg} \; ; \; h_{f5} = 561.4 \text{ kJ/kg.}$$

(i) **The steam bled per kg of steam supplied to the turbine, m :**

Considering the *energy balance for feed heater* we have ;

$$m(h_2' - h_{f5}) = (1 - m)(h_{f5} - h_{f4})$$

or $$m(2697 - 561.4) = (1 - m)(561.4 - 163.4)$$

or $$2135.6\, m = 398\,(1 - m)$$

∴ m = **0.157 kJ/kg of steam generated. (Ans.)**

(*ii*) **Steam generated per hour :**

Work developed per kg of steam in the turbine

$$= 1(h_1 - h_2') + (1 - m)(h_2' - h_3')$$

$$= (3300 - 2697) + (1 - 0.157)(2697 - 2234) = 993.3 \text{ kJ/kg}$$

Actual work developed by the turbine

$$= \frac{22500}{\eta_{alt.} \times \eta_{mech.}} = \frac{22500}{0.94 \times 0.97} = 24676.5 \text{ kW}$$

$\therefore$ Steam generated per hour $= \frac{24676.5}{993.3} \times \frac{3600}{1000}$ tonnes/h = **89.43 tonnes/h. (Ans.)**

(*iii*) **The overall efficiency of the plant, $\eta_{overall}$:**

Net power available deducting pump power

$$= 22500 (1 - 0.09) = 20475 \text{ kW}$$

Heat supplied in the boiler $= \frac{89.43 \times 1000 (h_1 - h_{f5})}{0.86}$ kJ/h

$$= \frac{89.43 \times 1000 (3300 - 561.4)}{0.86 \times 3600} \text{ kW} = 79106.3 \text{ kW}$$

$$\therefore \qquad \eta_{overall} = \frac{\text{Net power available}}{\text{Heat supplied by the boiler}}$$

$$= \frac{20475}{79106.3} = 0.2588$$ or **25.88%. (Ans.)**

Example 12.24. *A steam power plant of 110 MW capacity is equipped with regenerative as well as reheat arrangement. The steam is supplied at 80 bar and 55°C of superheat. The steam is extracted at 7 bar for feed heating and remaining steam is reheated to 350°C, and then expanded to 0.4 bar in the L.P. stage. Assume indirect type of feed heaters. Determine :*

(*i*) *The ratio of steam bled to steam generated,*

(*ii*) *The boiler generating capacity in tonnes of steam / hour, and*

(*iii*) *Thermal efficiency of the cycle.*

Assume no losses and ideal processes of expansion.

Solution. The schematic arrangement of the plant is shown in Fig. 12.39 (*a*) and the processes are represented on *h-s* chart in Fig. 12.39 (*b*).

Given : Capacity of plant = 110 MW ;

$$t_1 = 350°\text{C } i.e., t_s \text{ at 80 bar} \simeq 295°\text{C} + 55°\text{C} = 350°\text{C})$$

$$p_2 = p_3 = 7 \text{ bar} ; t_3 = 350°\text{C} ; p_4 = 0.4 \text{ bar}$$

- Locate point 1 corresponding to the condition $p_1 = 80$ bar and $t_1 = 350°$C, on the *h-s* chart.
- Locate point 2 by drawing vertical line through point 1 till it cuts the 7 bar pressure line.
- Locate point 3 as the cross point of 7 bar and 350°C temperature line.
- Locate point 4 by drawing vertical line through the point 3 till it cuts the 0.4 bar pressure line.

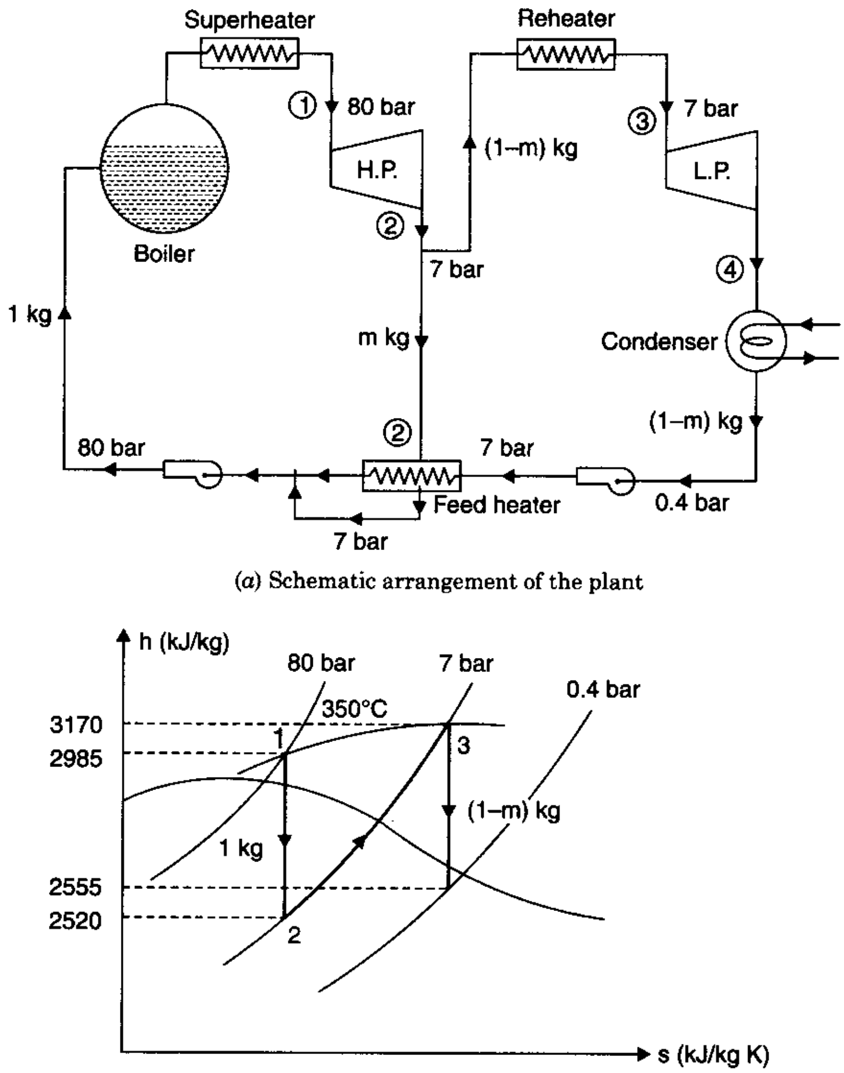
From *h-s* chart, we find :

$$h_1 = 2985 \text{ kJ/kg} ; h_2 = 2520 \text{ kJ/kg} ;$$

$$h_3 = 3170 \text{ kJ/kg} ; h_4 = 2555 \text{ kJ/kg}.$$

Also, from steam tables, we have :

$$h_{f2} \text{ (at 7 bar)} = 697.1 \text{ kJ/kg} ; h_{f4} \text{ (at 0.4 bar)} = 317.7 \text{ kJ/kg}.$$



(*a*) Schematic arrangement of the plant

(*b*) *h-s* diagram

Fig. 12.39

(*i*) **The ratio of steam bled to steam generated :**

Consider energy/heat balance of feed heater :

Heat lost by m kg of steam = Heat gained by $(1 - m)$ kg of condensed steam

$$m(h_2 - h_{f2}) = (1 - m)(h_{f2} - h_{f4})$$

$$m(2520 - 697.1) = (1 - m)(697.1 - 317.7)$$

$$1822.9\ m = (1 - m) \times 379.4$$

$$\therefore \quad m = 0.172 \text{ kg}$$

i.e. Amount of steam bled per kg of steam supplied to the turbine = 0.172 kg

$$\therefore \quad \frac{\text{Steam generated}}{\text{Steam bled}} = \frac{1}{0.172} = \mathbf{5.814.} \quad \textbf{(Ans.)}$$

(*ii*) **The boiler generating capacity :**

If m_s is the mass of steam supplied to the power plant per second, then the work developed is given by :

$$m_s(h_1 - h_2) + m_s(1 - m)(h_3 - h_4) = 110 \times 10^3$$

or, $$m_s(2985 - 2520) + m_s(1 - 0.172)(3170 - 2555) = 110 \times 10^3$$

or, $$m_s(465 + 509.22) = 110 \times 10^3$$

$\therefore$ $m_s = 112.91$ kg/s or **406.48 tonnes/hour (Ans.)**

(*iii*) **Thermal efficiency of the cycle, $\eta_{thermal}$:**

$$\eta_{thermal} = \frac{\text{Output/kg of steam}}{\text{Input/kg of steam}} = \frac{(h_1 - h_2) + (1 - m)(h_3 - h_4)}{(h_1 - h_{f_2}) + (1 - m)(h_3 - h_2)}$$

$$= \frac{(2985 - 2520) + (1 - 0.172)(3170 - 2555)}{(2985 - 697.1) + (1 - 0.172)(3170 - 2520)}$$

$$= \frac{974.22}{2826.1} = 0.3447 \quad \text{or} \quad \mathbf{34.47\%.} \quad \textbf{(Ans.)}$$

Example 12.25. *A steam power plant equipped with regenerative as well as reheat arrangement is supplied with steam to the H.P. turbine at 80 bar 470°C. For feed heating, a part of steam is extracted at 7 bar and remainder of the steam is reheated to 350°C in a reheater and then expanded in L.P. turbine down to 0.035 bar. Determine :*

(*i*) *Amount of steam bled-off for feed heating,*

(*ii*) *Amount of steam supplied to L.P. turbine,*

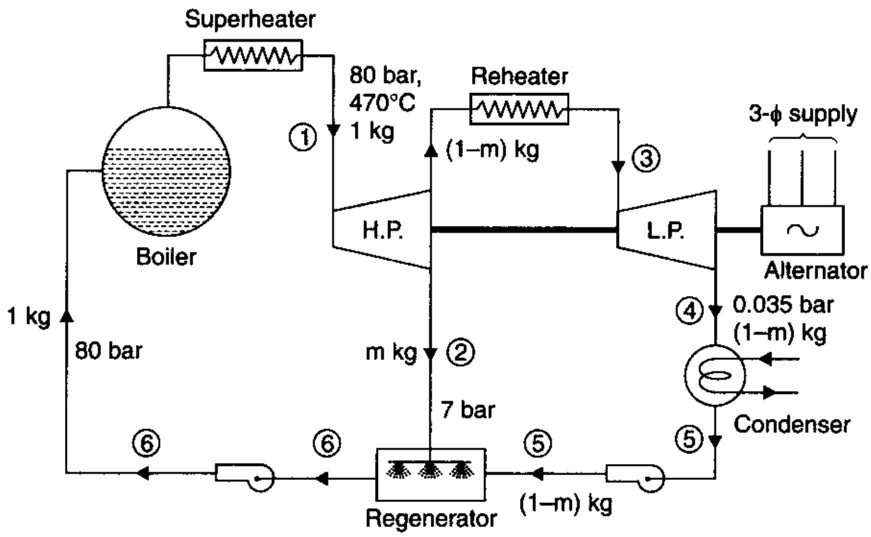
(*iii*) *Heat supplied in the boiler and reheater*

(*iv*) *Cycle efficiency, and*

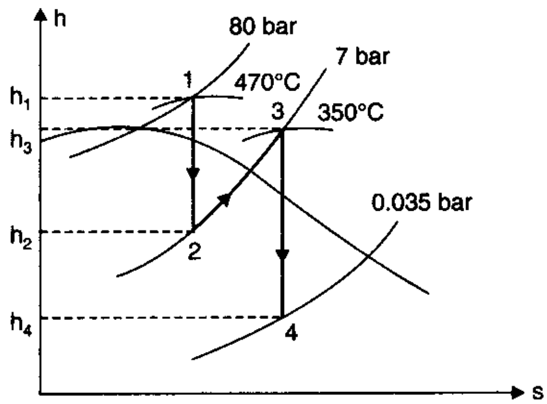
(*v*) *Power developed by the system.*

The steam supplied by the boiler is 50 kg/s. **(B.U. Dec., 2000)**

Solution. The schematic arrangement is the steam power plant of shown in Fig. 12.40 (*a*) and the processes are represented on *h-s* diagram as shown in Fig. 12.40 (*b*).



(*a*) Schematic arrangement of the steam power plant



(*b*) *h-s* diagram

Fig. 12.40

From *h-s* chart and steam tables, we have enthalpies at different points as follows :

h_1 = 3315 kJ/kg ; h_2 = 2716 kJ/kg
h_3 = 3165 kJ/kg ; h_4 = 2236 kJ/kg } From *h-s* chart

$h_{f6} = h_{f2}$ = 697.1 kJ/kg ; $h_{f5} = h_{f4}$ = 101.9 kJ/kg } From steam table.

(*i*) **Amount of steam bled off for feed heating :**

Considering *energy balance at regenerator, we have :*

Heat lost by steam = Heat gained by water

$$m(h_2 - h_{f6}) = (1 - m)(h_{f6} - h_{f5})$$

or $$m(h_2 - h_{f2}) = (1 - m)(h_{f2} - h_{f4}) \qquad [\because \; h_{f6} = h_{f2} \; ; h_{f5} = h_{f4}]$$

or $$m(2716 - 697.1) = (1 - m)(697.1 - 111.9)$$

or $$2018.9\, m = 585.2\,(1 - m)$$

$\therefore$ m = 0.225 g of steam supplied

Hence amount of **steam bled off is 22.5% of steam generated by the boiler. (Ans.)**

(*ii*) **Amount of steam supplied to L.P. turbine :**

Amount of steam supplied to L.P. turbine

= 100 – 22.5

= **77.5% of the steam generated by the boiler. (Ans.)**

(*iii*) **Heat supplied in the boiler and reheater**

Heat supplied in the *boiler* per kg of steam generated

$= h_1 - h_{f6}$ = 3315 – 697.1 = **2617.9 kJ/kg. (Ans.)**

$(\because \; h_{f6} = h_{f2})$

Heat supplied in the *reheater* per kg of steam generated

$= (1 - m)(h_3 - h_2)$

= (1 – 0.225) (3165 – 2716) = **347.97 kJ/kg. (Ans.)**

Total amount of heat supplied by the boiler and reheater per kg of steam generated,

$$Q_s = 2617.9 + 347.97 = 2965.87 \text{ kJ/kg}$$

(*iv*) **Cycle efficiency, η_{cycle} :**

Amount of work done by per kg of steam generated by the boiler,

$$W = 1(h_1 - h_2) + (1 - m)(h_3 - h_4), \text{ Neglecting pump work}$$
$$= (3315 - 2716) + (1 - 0.225)(3165 - 2236) = 1319 \text{ kJ/kg}$$

$$\therefore \quad \eta_{cycle} = \frac{W}{Q_s} = \frac{1319}{2965.87} = 0.4447 \quad \text{or} \quad \mathbf{44.47\%} \quad \textbf{(Ans.)}$$

(*v*) **Power developed by the system :**

Power developed by the system

$$= m_s \times W = 50 \times 1319 \text{ kJ/s} = \frac{50 \times 1319}{1000}$$
$$= \mathbf{65.95\ MW} \quad \textbf{(Ans.)}$$

Example 12.26. *A steam power plant operates on ideal Rankine cycle using reheater and regenerative feed water heaters. It has one open feed heater. Steam is supplied at 150 bar and 600°C. The condenser pressure is 0.1 bar. Some steam is extracted from the turbine at 40 bar for closed feed water heater and remaining steam is reduced at 40 bar to 600°C. Extracted steam is completely condensed in this closed feed water heater and is pumped to 150 bar before mixing with the feed water heater. Steam for the open feed water heater is bled from L.P. turbine at 5 bar. Determine :*

(*i*) *Fraction of steam extracted from the turbines at each bled heater, and*

(*ii*) *Thermal efficiency of the system.*

Draw the line diagram of the components and represent the cycle on T-s diagram.

(P.U. Dec., 2001)

Solution. The arrangement of the components is shown in Fig. 12.41 (*a*) and the processes are represented on *T-s* diagram as shown in Fig. 12.41 (*b*).

From *h-s* chart and steam tables we have enthalpies at different points as follows :

$$\left.\begin{aligned} &h_1 = 3578 \text{ kJ/kg} ; \quad h_2 = 3140 \text{ kJ/kg} ; \\ &h_3 = 3678 \text{ kJ/kg} ; \quad h_4 = 3000 \text{ kJ/kg} ; \\ &h_5 = 2330 \text{ kJ/kg} ; \end{aligned}\right\} \text{From } h\text{-}s \text{ chart}$$

$$\left.\begin{aligned} &h_{f1} \text{ (at 150 bar)} = 1611 \text{ kJ/kg} \\ &h_{f2} \text{ (at 40 bar)} = 1087.4 \text{ kJ/kg} ; h_{f4} \text{ (at 5 bar)} = 640.1 \text{ kJ/kg} ; \\ &h_{f5} = h_{f6} \text{ (at 0.1 bar)} = 191.8 \text{ kJ/kg} \end{aligned}\right\} \text{Steam tables}$$

(*i*) **Fraction of steam extracted from the turbines at each bled heater m_1, m_2 :**

Considering *energy balance for closed feed heater,* we have :

$$m_1(h_2 - h_{f2}) = (1 - m_1)(h_{f2} - h_{f4})$$
$$m_1(3140 - 1087.4) = (1 - m_1)(1087.4 - 640.1)$$

or $$2052.6\, m_1 = (1 - m_1) \times 447.3$$

$\therefore$ $\mathbf{m_1 = 0.179}$ **kg/kg of steam supplied by the boiler. (Ans.)**

Considering *energy balance for open feed heater,* we have :

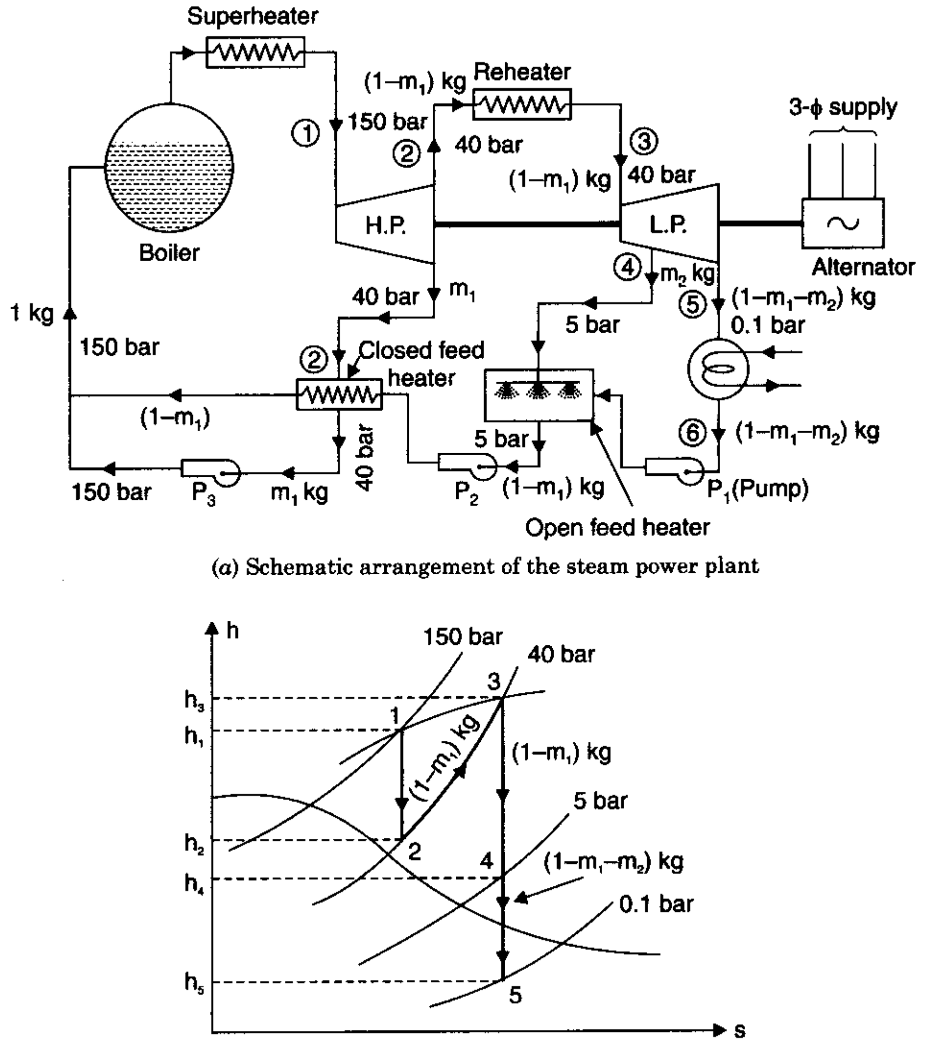
$$m_2(h_4 - h_{f4}) = (1 - m_1 - m_2)(h_{f4} - h_{f6})$$

or $$m_2(h_4 - h_{f4}) = (1 - m_1 - m_2)(h_{f4} - h_{f5}) \quad (\because\ h_{f6} = h_{f5})$$

or $$m_2(3000 - 640.1) = (1 - 0.179 - m_2)(640.1 - 191.8)$$

or $$2359.9\, m_2 = (0.821 - m_2) \times 448.3 = 368.05 - 448.3\, m_2$$

$\therefore$ $\mathbf{m_2 = 0.131}$ **kg/kg of steam supplied by boiler. (Ans.)**



(*a*) Schematic arrangement of the steam power plant

(*b*) *h-s* diagram

Fig. 12.41

(*ii*) **Thermal efficiency of the system, $\eta_{thermal}$:**

Total work done per kg of steam supplied by the boiler

$$= 1 \times (h_1 - h_2) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$$

$$= (3578 - 3140) + (1 - 0.179)(3678 - 3000) + (1 - 0.179 - 0.131)(3000 - 2330)$$

$$= 438 + 556.64 + 462.3 = 1456.94 \text{ kJ/kg}$$

Work done by the pump P_1

$$W_{P1} = v_{w1}(1 - m_1 - m_2)(5 - 0.1) \times 10^5 \times 10^{-3} \text{ kJ/kg}$$

$$= \frac{1}{1000}(1 - 0.179 - 0.131)(5 - 0.1) \times 10^5 \times 10^{-3} = 0.338 \text{ kJ/kg}$$

$$\left[\text{Taking } v_{w1} = v_{w2} = v_{w3} = \frac{1}{1000} \text{ m}^3/\text{kg}\right]$$

Work done by the pump P_2,

$$W_{P2} = v_{w2}\,(1 - m_1)(150 - 5) \times 10^5 \times 10^{-3} \text{ kJ/kg}$$

$$= \frac{1}{1000}\,(1 - 0.179)(150 - 5) \times 10^5 \times 10^{-3} = 11.9 \text{ kJ/kg}$$

Work done by pump P_3,

$$W_{P3} = v_{w3} \times m_1 \times (150 - 40) \times 10^5 \times 10^{-3}$$

$$= \frac{1}{1000} \times 0.179\,(150 - 40) \times 10^5 \times 10^{-3} = 1.97 \text{ kJ/kg}$$

Total pump work $= W_{P1} + W_{P2} + W_{P3}$

$= 0.338 + 11.9 + 1.97 = 14.21$ kJ/kg of steam supplied by boiler

$\therefore$ Net work done by the turbine per kg of steam supplied by the boiler,

$$W_{net} = 1456.94 - 14.21 = 1442.73 \text{ kJ/kg}$$

Heat of feed water extering the boiler

$$= (1 - m_1) \times 1611 + m_1 \times 1611 = 1611 \text{ kJ/kg}$$

Heat supplied by the boiler per kg of steam,

$$Q_{s1} = h_1 - 1610 = 3578 - 1610 = 1968 \text{ kJ/kg}$$

Q_{s2} = Heat supplied in the reheater

$$= (1 - m_1)(h_3 - h_2) = (1 - 0.179)(3678 - 3140)$$

$= 441.7$ kJ/kg of steam supplied by the boiler

Q_{st} (Total heat supplied) $= Q_{s1} + Q_{s2} = 1968 + 441.7 = 2409.7$ kJ/kg

$$\therefore \quad \eta_{thermal} = \frac{W_{net}}{Q_{st}} = \frac{1442.73}{2409.7} = 0.5987 \text{ or } \mathbf{59.87\%.} \quad \textbf{(Ans.)}$$

Example 12.27. *Steam at 70 bar and 450°C is supplied to a steam turbine. After expanding to 25 bar in high pressure stages, it is reheated to 420°C at the constant pressure. Next ; it is expanded in intermediate pressure stages to an appropriate minimum pressure such that part of the steam bled at this pressure heats the feed water to a temperature of 180°C. The remaining steam expands from this pressure to a condenser pressure of 0.07 bar in the low pressure stage. The isentropic efficiency of H.P. stage is 78.5%, while that of the intermediate and L.P. stages is 83% each. From the above data, determine :*

(*i*) *The minimum pressure at which bleeding is necessary.*

(*ii*) *The quantity of steam bled per kg of flow at the turbine inlet.*

(*iii*) *The cycle efficiency.*

Neglect pump work. **(Roorkee University)**

Solution. The schematic arrangement of the plant is shown in Fig. 12.42 (*a*) and the processes are represented on *T-s* and *h-s* diagrams as shown in Figs. 12.42 (*b*) and (*c*) respectively.

(*i*) **The minimum pressure at which bleeding is necessary :**

It would be assumed that the feed water heater is an open heater. Feed water is heated to 180°C. So p_{sat} at 180°C $\simeq$ 10 bar is the pressure at which the heater operates.

Thus, *the pressure at which bleeding is necessary is* **10 bar. (Ans.)**

From the *h-s* chart (Mollier chart), we have :

$$h_1 = 3285 \text{ kJ/kg} ;\ h_2 = 2980 \text{ kJ/kg} ;\ h_3 = 3280 \text{ kJ/kg} ;\ h_4 = 3030 \text{ kJ/kg}$$

$$h_3 - h_4' = 0.83(h_3 - h_4) = 0.83(3280 - 3030) = 207.5 \text{ kJ/kg}$$

$$\therefore \quad h_4' = h_3 - 207.5 = 3280 - 207.5 = 3072.5 \text{ kJ/kg}$$

$$h_5 = 2210 \text{ kJ/kg}$$

$$h_4' - h_5' = 0.83(h_4' - h_5) = 0.83(3072.5 - 2210) \simeq 715.9 \text{ kJ/kg}$$

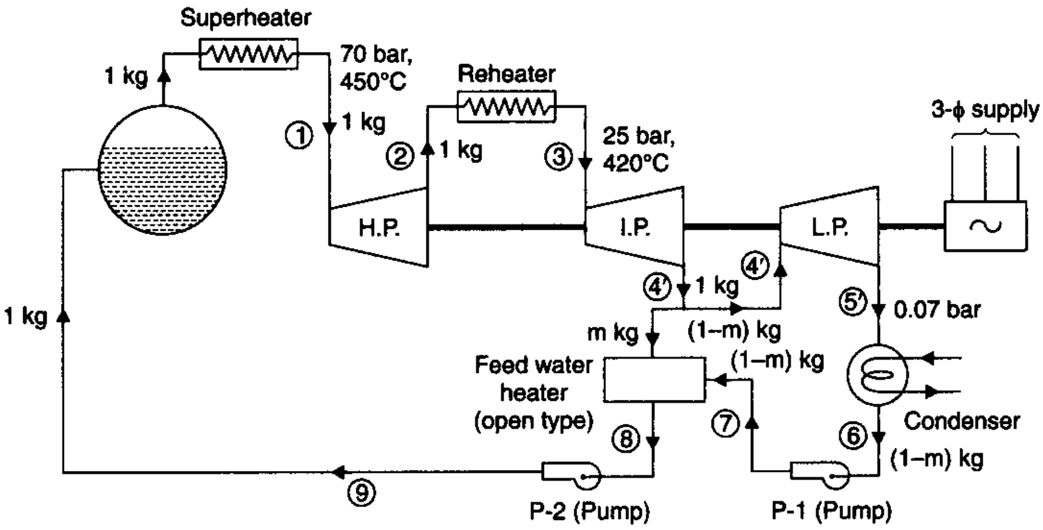
$$\therefore \quad h_5' = h_4' - 715.9 = 3072.5 - 715.9 = 2356.6 \text{ kJ/kg}$$

From steam tables, we have :

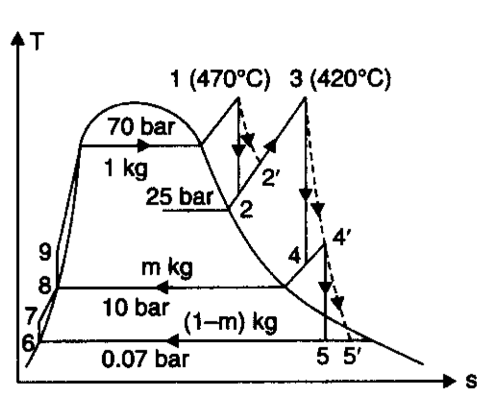
$$h_{f6} = 163.4 \text{ kJ/kg} \; ; \quad h_{f8} = 762.6 \text{ kJ/kg}$$

$$h_1 - h_2' = 0.785(h_1 - h_2) = 0.785(3285 - 2980) = 239.4 \text{ kJ/kg}$$

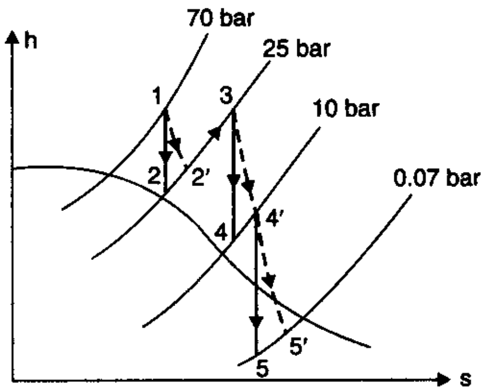
$$\therefore \quad h_2' = h_1 - 239.4 = 3285 - 239.4 = 3045.6 \text{ kJ/kg}$$



(a) Schematic arrangement of the plant



(b) T-s diagram



(c) h-s diagram (Pump work not shown)

Fig. 12.42

(ii) **The quantity of steam bled per kg of flow at the turbine inlet, m :**

Considering *energy balance for the feed water heater, we have :*

$$m \times h_4' + (1 - m)\, h_{f7} = 1 \times h_{f8}$$

$$m \times 3072.5 + (1 - m) \times 163.4 = 1 \times 762.6 \qquad (\because \; h_{f7} = h_{f6})$$

$$3072.5\, m + 163.4 - 163.4\, m = 762.6$$

$$\therefore \qquad m = \frac{(762.6 - 163.4)}{(3072.5 - 163.4)}$$

$= \mathbf{0.206}$ **kg of steam flow at turbine inlet. (Ans.)**

(*iii*) **Cycle efficiency,** η_{cycle} **:**

$$\eta_{cycle} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{1(h_1 - h_2) + 1(h_3 - h_4) + (1 - m)(h_4' - h_5')}{(h_1 - h_{f8}) + (h_3 - h_2')}$$

$$= \frac{(3285 - 3045.6) + 207.5 + (1 - 0.206)(715.9)}{(3285 - 762.6) + (3280 - 3045.6)} = \frac{1015.3}{2756.8}$$

$= 0.3683$ or **36.83%. (Ans.)**

HIGHLIGHTS

1. Carnot cycle efficiency $= \dfrac{T_1 - T_2}{T_1}$.
2. Rankine cycle is the theoretical cycle on which steam primemovers work.

   Rankine efficiency $= \dfrac{h_1 - h_2}{h_1 - h_{f_2}}$.
3. The thermal efficiency of Rankine cycle is increased by
   (*i*) *Increasing the average temperature at which heat is added to the cycle.*
   (*ii*) *Decreasing the average temperature at which heat is rejected to the cycle.*
4. Thermal efficiency of regenerative cycle

$$= \frac{(h_0 - h_1) + (1 - m_1)(h_1 - h_2) + (1 - m_1 - m_2)(h_2 - h_3)}{(h_0 - h_{f_6})} .$$

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. Rankine cycle efficiency of a good steam power plant may be in the range of
   (*a*) 15 to 20% (*b*) 35 to 45%
   (*c*) 70 to 80% (*d*) 90 to 95% .
2. Rankine cycle operating on low pressure limit of p_1 and high pressure limit of p_2
   (*a*) has higher thermal efficiency than the Carnot cycle operating between same pressure limits
   (*b*) has lower thermal efficiency than Carnot cycle operating between same pressure limits
   (*c*) has same thermal efficiency as Carnot cycle operating between same pressure limits
   (*d*) may be more or less depending upon the magnitudes of p_1 and p_2.
3. Rankine efficiency of a steam power plant
   (*a*) improves in summer as compared to that in winter
   (*b*) improves in winter as compared to that in summer
   (*c*) is unaffected by climatic conditions
   (*d*) none of the above.
4. Rankine cycle comprises of
   (*a*) two isentropic processes and two constant volume processes
   (*b*) two isentropic processes and two constant pressure processes

(c) two isothermal processes and two constant pressure processes
(d) none of the above.

5. In Rankine cycle the work output from the turbine is given by
(a) change of internal energy between inlet and outlet
(b) change of enthalpy between inlet and outlet
(c) change of entropy between inlet and outlet
(d) change of temperature between inlet and outlet.

6. Regenerative heating *i.e.*, bleeding steam to reheat feed water to boiler
(a) decreases thermal efficiency of the cycle
(b) increases thermal efficiency of the cycle
(c) does not affect thermal efficiency of the cycle
(d) may increase or decrease thermal efficiency of the cycle depending upon the point of extraction of steam.

7. Regenerative cycle thermal efficiency
(a) is always greater than simple Rankine thermal efficiency
(b) is greater than simple Rankine cycle thermal efficiency only when steam is bled at particular pressure
(c) is same as simple Rankine cycle thermal efficiency
(d) is always less than simple Rankine cycle thermal efficiency.

8. In a regenerative feed heating cycle, the optimum value of the fraction of steam extracted for feed heating
(a) decreases with increase in Rankine cycle efficiency
(b) increases with increase in Rankine cycle efficiency
(c) is unaffected by increase in Rankine cycle efficiency
(d) none of the above.

9. In a regenerative feed heating cycle, the greatest economy is affected
(a) when steam is extracted from only one suitable point of steam turbine
(b) when steam is extracted from several places in different stages of steam turbine
(c) when steam is extracted only from the last stage of steam turbine
(d) when steam is extracted only from the first stage of steam turbine.

10. The maximum percentage gain in Regenerative feed heating cycle thermal efficiency
(a) increases with number of feed heaters increasing
(b) decreases with number of feed heaters increasing
(c) remains same unaffected by number of feed heaters
(d) none of the above.

Answers

1. (b)	2. (a)	3. (b)	4. (b)	5. (b)	6. (b)	7. (a)
8. (b)	9. (b)	10. (a).				

THEORETICAL QUESTIONS

1. Explain the various operation of a Carnot cycle. Also represent it on a *T-s* and *p-V* diagrams.
2. Describe the different operations of Rankine cycle. Derive also the expression for its efficiency.
3. State the methods of increasing the thermal efficiency of a Rankine cycle.
4. Explain with the help of neat diagram a 'Regenerative Cycle'. Derive also an expression for its thermal efficiency.
5. State the advantages of regenerative cycle/simple Rankine cycle.
6. Explain with a neat diagram the working of a Binary vapour cycle.

UNSOLVED EXAMPLES

1. A simple Rankine cycle works between pressure of 30 bar and 0.04 bar, the initial condition of steam being dry saturated, calculate the cycle efficiency, work ratio and specific steam consumption.
[**Ans.** 35%, 0.997, 3.84 kg/kWh]
2. A steam power plant works between 40 bar and 0.05 bar. If the steam supplied is dry saturated and the cycle of operation is Rankine, find :
(*i*) Cycle efficiency (*ii*) Specific steam consumption.
[**Ans.** (*i*) 35.5%, (*ii*) 3.8 kg/kWh]
3. Compare the Rankine efficiency of a high pressure plant operating from 80 bar and 400°C and a low pressure plant operating from 40 bar 400°C, if the condenser pressure in both cases is 0.07 bar.
[**Ans.** 0.391 and 0.357]
4. A steam power plant working on Rankine cycle has the range of operation from 40 bar dry saturated to 0.05 bar. Determine :
(*i*) The cycle efficiency (*ii*) Work ratio
(*iii*) Specific fuel consumption. [**Ans.** (*i*) 34.64%, (*ii*) 0.9957, (*iii*) 3.8 kg/kWh]
5. In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 30 bar and the exhaust pressure is 0.25 bar. Determine :
(*i*) The pump work (*ii*) Turbine work
(*iii*) Rankine efficiency (*iv*) Condenser heat flow
(*v*) Dryness at the end of expansion.
Assume flow rate of 10 kg/s. [**Ans.** (*i*) 30 kW, (*ii*) 7410 kW, (*iii*) 29.2%, (*iv*) 17900 kW, (*v*) 0.763]
6. In a regenerative cycle the inlet conditions are 40 bar and 400°C. Steam is bled at 10 bar in regenerative heating. The exit pressure is 0.8 bar. Neglecting pump work determine the efficiency of the cycle.
[**Ans.** 0.296]
7. A turbine with one bleeding for regenerative heating of feed water is admitted with steam having enthalpy of 3200 kJ/kg and the exhausted steam has an enthalpy of 2200 kJ/kg. The ideal regenerative feed water heater is fed with 11350 kg/h of bled steam at 3.5 bar (whose enthalpy is 2600 kJ/h). The feed water (condensate from the condenser) with an enthalpy of 134 kJ/kg is pumped to the heater. It leaves the heater dry saturated at 3.5 bar. Determine the power developed by the turbine. [**Ans.** 16015 kW]
8. A binary-vapour cycle operates on mercury and steam. Saturated mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhaust at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar.
(*i*) Find the overall efficiency of the cycle.
(*ii*) If 50000 kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine ?
(*iii*) Assuming that all processes are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow ?
(*iv*) If the steam leaving the mercury condenser is superheated to a temperature of 300°C in a superheater located in the mercury boiler, and if the internal efficiencies of the mercury and steam turbines are 0.85 and 0.87 respectively, calculate the overall efficiency of the cycle. The properties of saturated mercury are given below :

p (bar)	*t (°C)*	h_f (kJ/kg)	h_g	s_f (kJ/kg K)	s_g	v_f (m³/kg)	v_g
4.5	450	63.93	355.98	0.1352	0.5397	79.9×10^{-6}	0.068
0.04	216.9	29.98	329.85	0.0808	0.6925	76.5×10^{-3}	5.178

[**Ans.** (*i*) 52.94%, (*ii*) 59.35×10^4 kg/h, (*iii*) 28.49 MW, (*iv*) 46.2%]

13

Gas Power Cycles

13.1. DEFINITION OF A CYCLE

A **cycle** is defined as a *repeated series of operations occurring in a certain order*. It may be repeated by repeating the processes in the same order. The cycle may be of *imaginary perfect engine or actual engine*. The former is called ***ideal cycle*** and the latter ***actual cycle***. *In ideal cycle all accidental heat losses are prevented and the working substance is assumed to behave like a perfect working substance.*

13.2. AIR STANDARD EFFICIENCY

To compare the effects of different cycles, it is of paramount importance that the effect of the calorific value of the fuel is altogether eliminated and this can be achieved by considering air (which is assumed to behave as a perfect gas) as the working substance in the engine cylinder. *The efficiency of engine using air as the working medium is known as an "**Air standard efficiency**"*. This efficiency is oftenly called ***ideal efficiency***.

The actual efficiency of a cycle is always *less* than the air-standard efficiency of that cycle under ideal conditions. This is taken into account by introducing a new term "***Relative efficiency***" which is defined as :

$$\eta_{relative} = \frac{\text{Actual thermal efficiency}}{\text{Air standard efficiency}} \quad ...(13.1)$$

The analysis of all air standard cycles is based upon the following *assumptions* :

Assumptions :

1. The gas in the engine cylinder is a *perfect gas i.e.*, it obeys the gas laws and has constant specific heats.
2. The physical constants of the gas in the cylinder are the same as those of air at moderate temperatures *i.e.*, the molecular weight of cylinder gas is 29.

$$c_p = 1.005 \text{ kJ/kg-K}, \ c_v = 0.718 \text{ kJ/kg-K}.$$

3. The compression and expansion processes are adiabatic and they take place without internal friction, *i.e.*, these processes are *isentropic*.
4. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with cylinder at appropriate points during the process.